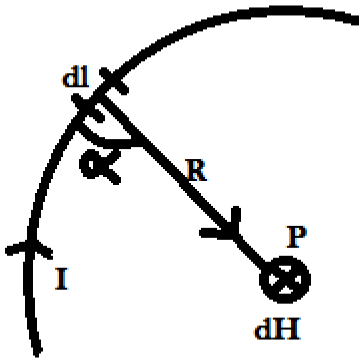
MAGNETO STATICS

Biot- Savart's law:-

Assume a current carrying conductor carries a constant current of 'I' amp. As shown in fig. below.

Biot Savart's law states that the magnetic field intensity 'dH' produced at a point p, by the differential current element Id*l* is proportional to the product Id*l* and the sine of the angle 'α' between the element and is inversely proportional to the square of the distance 'R' between p and the element.



That is $dH \propto Idl, \sin\alpha, \frac{1}{R^2}$

$$dH \propto \frac{Idl \sin \alpha}{R^2}; \qquad dH = \frac{K\, Idl \sin \alpha}{R^2} \longrightarrow 1$$

Where k is the proportionality constant. In SI units, K=$\frac{1}{4\pi}$

$\because$ eqn. 1 becomes as

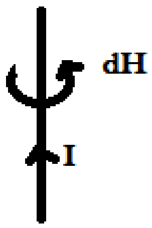
$$dH = \frac{Idl \sin \alpha}{4\pi R^2} \longrightarrow 2$$

From the definition of the cross product above equation is put in vector form as

$$dH = \frac{Idl\ X\ i_R}{4\pi R^2}; \qquad \text{Where} \qquad \left(i_R = \frac{\vec{R}}{|\vec{R}|}\right)$$

$$dH = \frac{Idl\ X\ \vec{R}}{4\pi R^3} \longrightarrow \quad 3$$

The direction of 'dH' can be determined by the right hand rule, with the right hand thumb pointing in the direction of the current, the right hand fingers encircling the wire in the direction of dH.



Current elements have no separate existense. All elements making up the complete current filament contribute to H and must be included. The summation leads to the integral form of Biot saverts law:

$$H = \oint \frac{Idl\ X\ i_R}{4\pi R^2}$$

The magneticfield intensity is a vector quantity and its units are Ampere/meter.

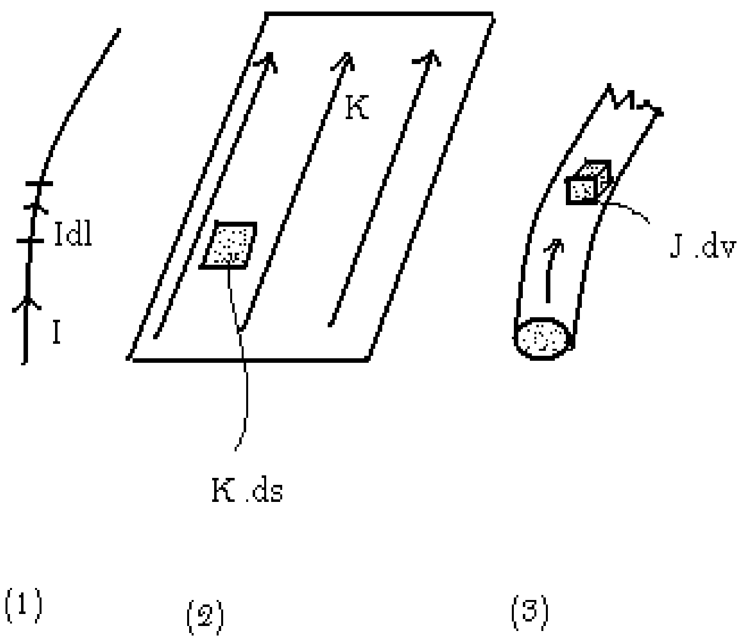
Current distributions:-

We have three different types of current distributions they are

Line current

Surface current

Volume current



distributions

If we define 'K' as the surface current density in amperes/meter and 'J' as the volume current density in amperes/$(\text{meter})^2$ then the source elements are related as

$$I\,dl = k\,ds = J\,dv \longrightarrow \quad 1$$

Thus in terms of the distributed current sources, the Biot savart's law becomes as

$$H = \int_L \frac{\mathrm{Idl\ X\ i_R}}{4\pi \mathrm{R^2}} \longrightarrow \quad 2$$

$$H = \int_S \frac{\mathrm{KdS\ X\ i_R}}{4\pi \mathrm{R^2}} \longrightarrow \quad 3$$ (surface current)

$$H = \int_V \frac{\mathrm{JdV\ X\ i_R}}{4\pi \mathrm{R^2}} \longrightarrow \quad 4$$ (volume current)

" An electrostatic field is produced by static (or) stationary charges. If the charges are moving with constant velocity, a static magnetic field is produced, i.e a magneto static field is produced by a constant current flow"

The two major law's of magnetostatic fields are

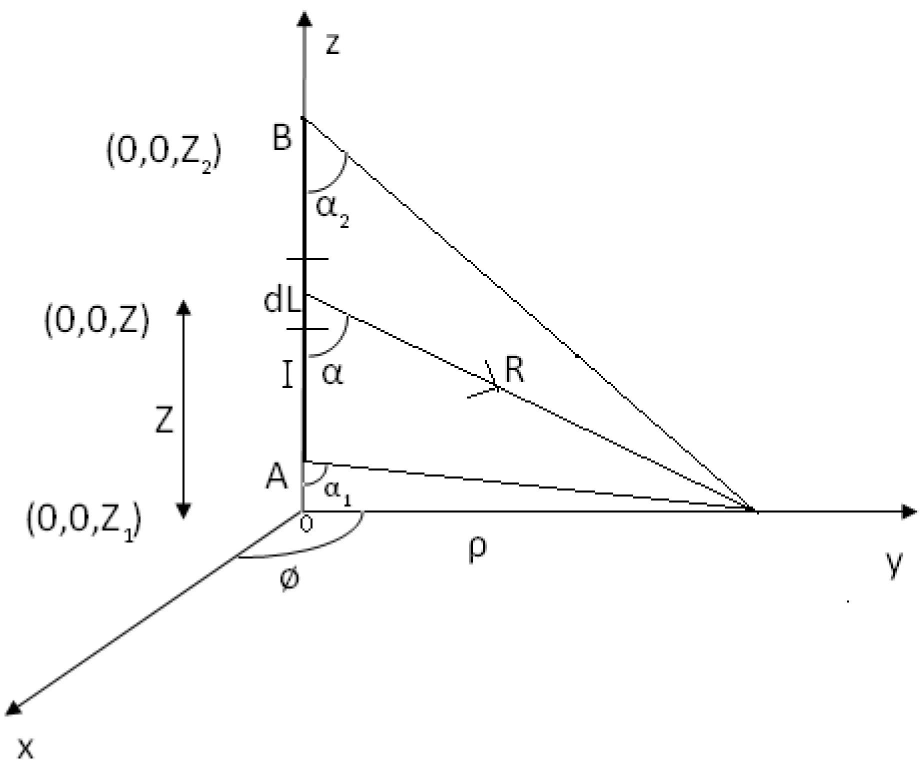
Biot – savart's law.

Ampere's circuit law.

Magnetic field due to a straight finite current carrying cinductor:-

consider a staright finite current carrying conductor of length AB as shown in the figure.

Assume that the conductor is along the z-axis with its upper and lower limits respectively subtending angles α_2 and α_1 at P i.e the point at which H is to be determined.



From Biot – savart's law the field intensity at P due to the differential current elelment Idl at (0,0,z) is given by

$$dH = \frac{\mathrm{Idl\ X\ \vec{R}}}{4\pi \mathrm{R}^3}$$

$$\vec{R} = \rho\, i_\rho - z\, i_z$$

$$|\vec{R}| = \sqrt{\rho^2 + z^2}$$

$$i_R = \frac{\vec{R}}{|\vec{R}|} = \frac{\rho\, i_\rho - z\, i_z}{\sqrt{\rho^2 + z^2}}$$

$$\mathrm{dL} = \mathrm{d_z\, i_z}$$

$$dH = \frac{I\mathrm{d_z\, i_z} \times \rho\, i_\rho - z\, i_z}{4\pi(\rho^2 + z^2)^{\frac{3}{2}}}$$

$$(\mathrm{d_z\, i_z}) \times \left(\rho\, i_\rho - z\, i_z\right) = \begin{vmatrix} i_\rho & i_\phi & i_z \\ 0 & 0 & \mathrm{dz} \\ \rho & 0 & -\mathrm{z} \end{vmatrix}$$

$$= -i_\phi\, [-\rho\, d_z]$$

$$= \rho\, d_z i_\phi$$

$$dH = \frac{I\, \rho\, d_z i_\phi}{4\pi(\rho^2 + z^2)^{\frac{3}{2}}}$$

Hence $H = \int_{z_1}^{z_2} \frac{I\, \rho\, d_z}{4\pi(\rho^2 + z^2)^{\frac{3}{2}}}\; i_\phi$

Let $z = \rho \cot \alpha$

$$dz = -\rho cos^2 \alpha\, d\alpha$$

$$H = \int_{\alpha_1}^{\alpha_2} \frac{I\,\rho\,(-\rho cosec^2\alpha)\,d\alpha}{4\pi(\rho^2 + \rho^2 cot^2\alpha)^{\frac{3}{2}}} i_\phi$$

$$H = \int_{\alpha_1}^{\alpha_2} -\frac{I\,\rho^2\,cosec^2\alpha\,d\alpha}{4\pi\rho^3 cosec^3\alpha} i_\phi$$

$$H = \int_{\alpha_1}^{\alpha_2} -\frac{I\,d\alpha}{4\pi\rho\,cosec\alpha} i_\phi$$

$$H = -\frac{I}{4\pi\rho} \int_{\alpha_1}^{\alpha_2} sin\,\alpha\,d\alpha\,i_\phi$$

$$H = -\frac{I}{4\pi\rho} (-\cos\alpha)_{\alpha_1}^{\alpha_2} i_\phi$$

$$\boxed{H = \frac{1}{4\pi\rho}(cos\alpha_2 - cos\alpha_1)\,i_\phi} \longrightarrow \quad 1$$

This expression is the magnetic field intensity at any point P due to a straight filamentary conductor of finite length.

When the conductor is semi infinite with respect to p, so that point A is now at O(0,0,0) while B is at (0,0,∞), $\alpha_1 = 90^0, \alpha_2 = 0^0$

And the above eqation becomes as

$$\boxed{H = \frac{I}{4\pi\rho}\,i_\phi} \longrightarrow \quad 2$$

When the conductor is infinite in length, for this case, point A is at (0,0,-∞) while B is at (0,0,∞); $\alpha_1 = 180^0, \alpha_2 = 0^0$ then eqn. 1 becomes as

$$H = \frac{I}{4\pi\rho}[1 - (-1)]\, i_\phi$$

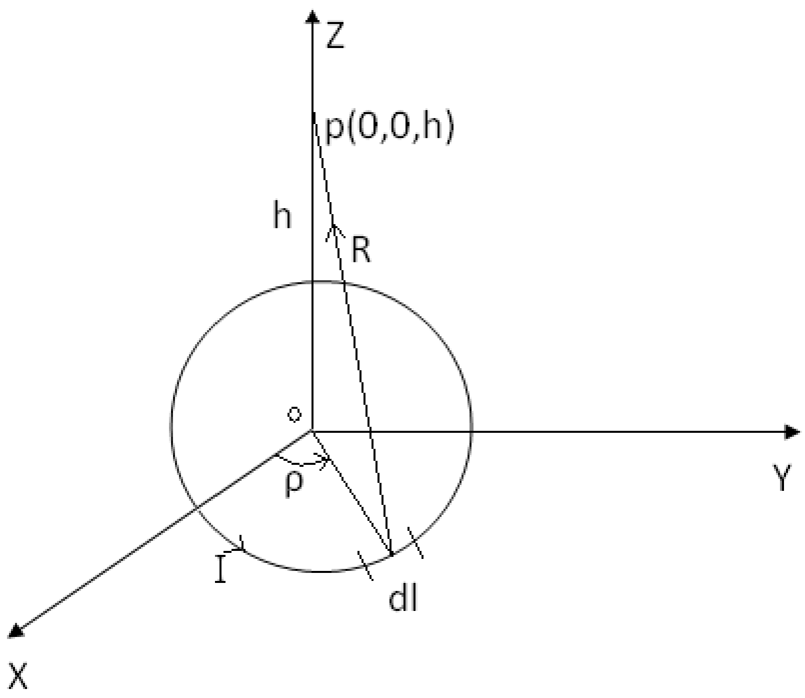
$$\boxed{H = \frac{I}{2\pi\rho}\, i_\phi}$$

A simple method to determine i_ϕ is

$$i_\phi = i_z \times i_\rho$$

Magnetic field intensity at a point on the axis of a circuar current loop:-

Consicer a circular loop as shown in figure below it carries a current of 'I' amperes along the i_{ϕ} direction



The magnetic field intensity at a point P(0,0,h) contributed by current element is given by Biot – savarts law

$$dH = \frac{Idl \times i_R}{4\pi R^2}$$

Where $dl = \rho \, d\phi \, i_{\phi}$

$$\vec{R} = -\rho i_{\rho} + h i_z$$

$$\therefore \quad dH = I\,\rho\, d\phi\, i_{\phi} \times (-\rho i_{\rho} + h i_{z})$$

Consider

$$\rho\, d\phi\, i_{\phi} \times (-\rho i_{\rho} + h i_{z}) = \begin{vmatrix} i_{\rho} & i_{\phi} & i_{z} \\ 0 & \rho d\phi & 0 \\ -\rho & 0 & h \end{vmatrix}$$

$$= i_{\rho}(\rho\, h\, d\phi) + i_{z}(\rho^{2} d\phi)$$

$$\therefore \qquad dH = \frac{I\,[\rho\, h\, d\phi\, i_{\rho} + \rho^{2} d\phi\, i_{z}]}{4\pi\,(\rho^{2} + h^{2})^{\frac{3}{2}}}$$

By symmetry the contribution along i_{ρ} add up to zero. Because the radial components produced by pairs of current element 180^{0} apart cancel.

$$\therefore \qquad dH = \frac{\mathrm{I}\,\rho^{2} d\phi\, i_{z}}{4\pi(\rho^{2} + h^{2})^{\frac{3}{2}}}$$

$$H = \int_{\phi=0}^{2\pi} \frac{\mathrm{I}\,\rho^{2} d\phi}{4\pi(\rho^{2} + h^{2})^{\frac{3}{2}}}\, i_{z}$$

$$H = \frac{\mathrm{I}\,\rho^{2}\, 2\pi}{4\pi(\rho^{2} + h^{2})^{\frac{3}{2}}}\, i_{z}$$

$$\boxed{H = \frac{\mathrm{I}\,\rho^{2}}{2(\rho^{2} + h^{2})^{\frac{3}{2}}}\, i_{z}\ {}^{A}/_{m}}$$

Example:1 A circular loop located on $x^2 + y^2 = 9,\ z = 0$ carries a direct current of 10A along i_ϕ determine H at (0,0,4) and (0,0,-4)

Soln: given $x^2 + y^2 = 9$

$\rho = 3$

$z = 0\ plane$

$I = 10A\ along\ i_\phi$

$$H = \frac{I\rho^2}{2(\rho^2 + h^2)^{\frac{3}{2}}} i_z\ A/m$$

At (0,0,4) ; h = 4

$$H = \frac{10 \times 9}{2(9+16)^{\frac{3}{2}}} i_z = 0.36\, i_z\, A/m$$

The magnetic field intensity is same for h=4 and h=-4 because it should not depend on h-value sign

Example:2 A thin ring of radius 5cm is placed on plane z = 1cm so that its center is at (0,0,1) If the ring carries 50mA along i_ϕ find H at (a) (0,0,-1cm) (b) (0,0,10cm)

Soln: given $\rho = 5cm$

$z = 1cm\, plane$

$O(0,0,1cm)$

$I = 50mA\ along\ i_\phi$

for H at (0,0,-1cm)

$h = 2cm$

$$H = \frac{50 \times 25}{2(25+4)^{\frac{3}{2}}}\ i_z$$

$$H = 400\, i_z\ milli \frac{A}{meter}$$

At (0,0,10cm)

$h = 9cm$

$$\therefore H = \frac{50 \times 25}{2(25+81)^{\frac{3}{2}}}\ i_z$$

$$H = 57.3\, i_z\ milli \frac{A}{meter}$$

The magnetic field intensity at the centre of the circular current loop is given by

$h = 0$

$$H = \frac{I\rho^2}{2\rho^3}\ i_z$$

$$\boxed{H = \frac{I}{2\rho}\ i_z\ A/m}$$

Ampere's circuit law

" Ampere's circuit law states that the line integral of the tangential component of H around a closed path is same as the net current (I_{enc}) enclosed by the path."

In other words circulation of H equals I_{enc}

$$\text{i.e} \quad \oint_L H.dl = I_{enclosed} \longrightarrow 1$$

ampere's law is useful in determining 'I' from 'H'

By applying stoke's theorem to the above equation we get

$$I_{enc} = \oint_L H.dl = \int_s (\nabla \times H). \longrightarrow 2$$

If I is the total current through the surface then

$$I_{enc} = \int_s J.ds \longrightarrow 3$$

By comparing eqns 2 & 3 we get

$$I_{enc} = \oint_L H.dl = \int_s (\nabla \times H) = \int_s J.ds$$

Comparing surface integrals we get

$$\boxed{\nabla \times H = J}$$

The units for $\nabla \times H$ is Ampere/meter

Applications of Ampere's law :

Conditions (or) procedure for applying (Gauss's law) Ampere's law

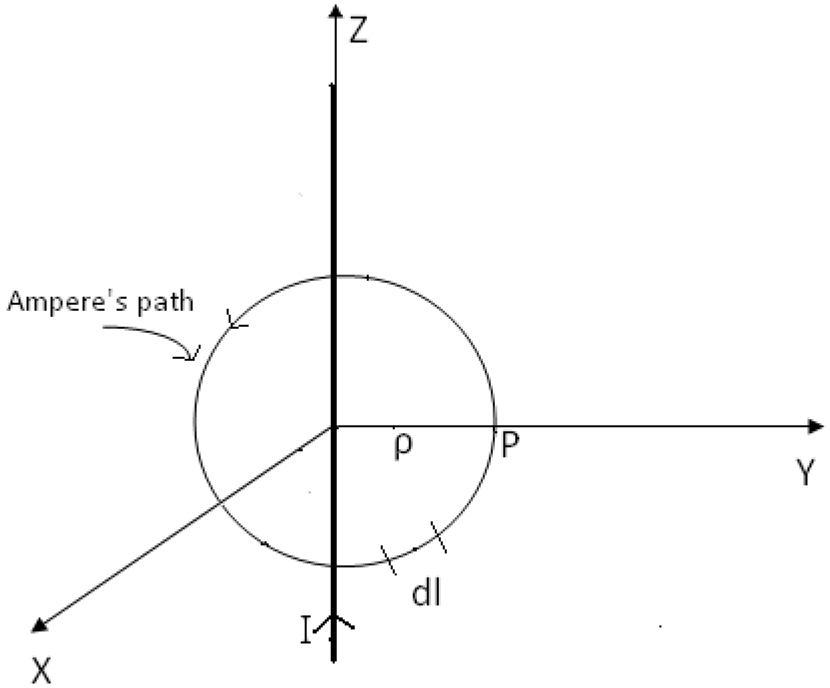
Symmetrical current distribution

Closed path (amperian path)

H is tangential to the closed path.

A.Infinite line current :

Consider an infinite long filamentary conductor carrying current 'I' along the Z-axis as shown in figure below



To determine 'H' at an observation point we allow a closed path pass through P. We choose a concentric circle as the amperian path which shows that 'H' is constant provided ρ is cinstant

This path encloses the whole current 'I', according to Ampere's law

$$I_{enc} = \oint_L H.dl$$

$$I_{enc} = \int H_\phi i_\phi . \rho d i_\phi$$

$$I_{enc} = \int \rho \, H_\phi d \, \phi$$

$$I_{enc} = \rho \, H_\phi \int_{\phi=0}^{2\pi} d\phi$$

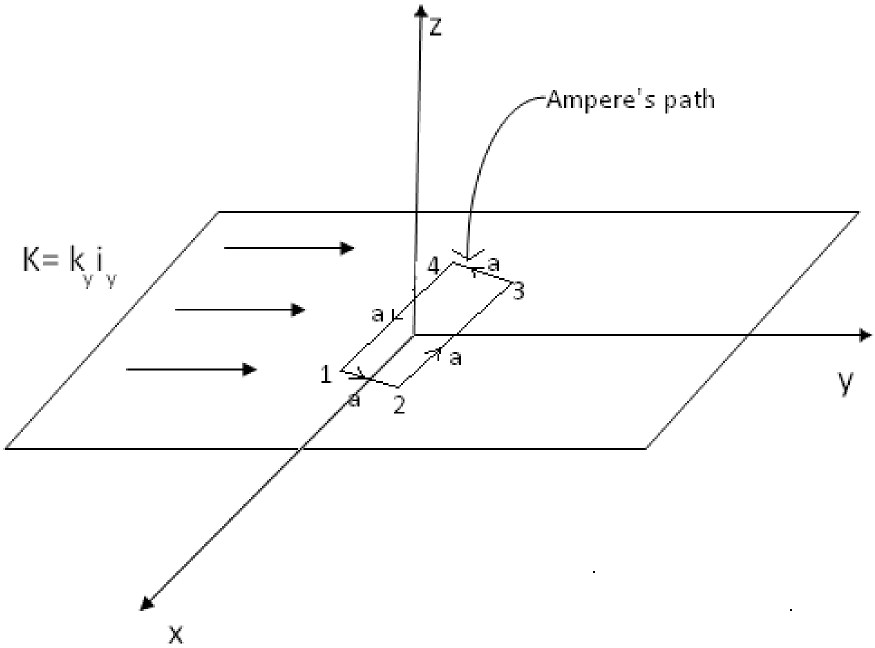
$$I = \rho \, H_\phi \, 2\pi$$

$$H_\phi = \frac{I}{2\pi\rho}$$

Hence $H = \frac{I}{2\pi\rho} i_\phi \; A/m$

B.Infinite sheet of current :

Consider an infinite current sheet in the z=0 plane and the sheet has a uniform current density $K = K_y \, i_y \; A/m$ as shown in figure below



Consider a closed path(Amperian path) 1-2-3-4-1 with centre P and each side as 'a'

The field can be thought of considering sheet as current elements.

No field will exist in 1-2 or3-4 directions H on one side of the sheet is the negative of that on the other side.

Applying Ampere's law to the rectangular closed path gives.

$$\oint_L H.dl = I_{enc}$$

$$\int_1^2 H.dl + \int_2^3 H.dl + \int_3^4 H.dl + \int_4^1 H.dl = K\,a$$

$$0 + (-H_x)(-a) + 0 + H_x(a) = K\,a$$

$$H_x\,a + H_x\,a = K\,a$$

$$2H_x\,a = K\,a$$

$$H_x = \frac{1}{2}\,K$$

Hence $H = \frac{1}{2} K\, i_x\ A/m$ Z > 0

$H = -\frac{1}{2} K\, i_x\ A/m$ Z < 0

In general, for an infinite sheet of current density K A/m

$$H = \frac{1}{2} K\, i_n\ A/m$$

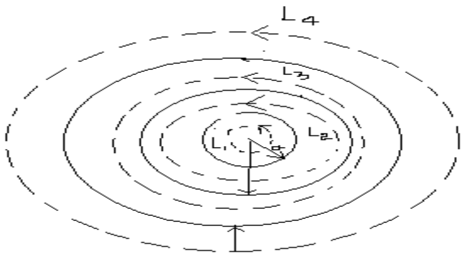
Where i_n is a unit normal vector directed from the current sheet to the point of interest.

C: Infinitely long coaxial cable;

Consider an infinitely long transmission line consisting of two concentric cylinders having their axes along the z-axis.

The inner conductor has radius 'a' and carrying current 'I', while the outer conductor has inner radius 'b' and thickness 't' and carries return current –I amp.

To determine 'H' everywhere assuming the current is uniformly distributed in both conductors.



Ca

se 1 >

For region $0 \le p \le a$

We apply Ampere's law to path L, giving

$\oint_{L_1} H.dl = I_{enclosed} = \int_s J.ds$ ⟶ 1

Since the current is uniformly distributed over the section.

$J = \dfrac{I}{\pi a^2}\ i_z$

$ds = \boldsymbol{\rho d\Phi d\rho} i_z$

$I_{enclosed} = \int_s J.ds \int_s \dfrac{I}{\pi a^2}\ i_z\, \boldsymbol{\rho d\Phi d\rho} i_z$

$I_{enclosed} = \int_s \dfrac{I}{\pi a^2}\ \boldsymbol{\rho d\Phi d\rho}$

$I_{enclosed} = \frac{I}{\pi a^2} [\ \frac{\rho^2}{2}\ ]^{\rho}{}_{0}\ [\ \boldsymbol{\Phi}\ ]^{2\Pi}{}_{0}$ ⟶ 2

$$I_{enclosed} = \frac{I\rho^2}{a^2}$$

$$\oint_{L_1} H.dl = \int H_{\Phi}\, I_{\Phi}\, \boldsymbol{\rho d\Phi i_{\Phi}}$$

$$= \int H_{\Phi}\ \boldsymbol{\rho d\Phi}$$

$$= \rho H_{\Phi}\, 2\Pi \longrightarrow \quad 3$$

Substituting equation 2 & 3 in equation 1

$$\rho H_{\Phi}\, 2\Pi = \frac{I\rho^2}{a^2}$$

$$\boxed{H_{\Phi} = \frac{I\rho}{2\Pi\, a^2}}$$

$$A/m \qquad 0 \leq \rho \leq a$$

Case 2>

For region $a \leq \rho \leq b$, we use path L_2 as the Ampere's path

$$\therefore \quad \oint_{L_1} H.dl = I_{enclosed} = \mathrm{I}$$

$$\int H_{\Phi}\, I_{\Phi}\, \boldsymbol{\rho d\Phi} i_{\Phi} = I$$

$$\boldsymbol{\rho} H_{\Phi}\, 2\Pi = I$$

$$\boxed{H_{\Phi} = \frac{I}{2\Pi\rho}}$$

$$A/m$$

$a \leq \rho \leq b.$

Since the whole current I enclosed by L_2 and it is independent of a.

Case 3>

→ for region $b \leq \rho \leq b + t.$

We apply Ampere's law to pathL_3, gives

$$\oint_{L_3} H.dl = I_{enclosed}$$

$$\rho H_\Phi 2\Pi = I_{enclosed} \longrightarrow \quad 1$$

→ Where $I_{enclosed} = I + \int J.ds$

J in this case is the current density of the outer conductor.

$$J = \frac{-I}{\Pi[(b+t)^2 - b^2]} i_z$$

$$I_{enclosed} = I - \int_0^{2\Pi} \int_0^{\rho} \frac{I}{\Pi[(b+t)^2 - b^2]} i_z \, \rho d\Phi i_\Phi i_z.$$

$$I_{enclosed} = I - \frac{I}{\pi((b+t)^2 - b^2)} \int_{\Phi=0}^{2\pi} d\Phi \int_{\rho=b}^{\rho} \rho d\rho$$

$$I_{enclosed} = I - \frac{I}{\pi(t^2 + 2bt)} \times 2\pi \times {[p^2 - b^2]}/{2}$$

$$I_{enclosed} = I - \frac{I[p^2 \; - b^2]}{(t^2 + 2bt)}$$

$$I_{enclosed} = I\left(1 - \frac{[p^2 \; - b^2]}{(t^2 + 2bt)}\right) \longrightarrow \quad 2$$

Substituting eqn (2) in eqn (1)

$$2\pi\rho H_{\Phi} = I\left(1 - \frac{[p^2 \; - b^2]}{(t^2 + 2bt)}\right)$$

$$\boxed{H_{\Phi} = \frac{I\left(1 - \frac{[p^2 \; - b^2]}{(t^2 + 2bt)}\right)}{2\pi\rho}} \; A/m \qquad b \leq \rho \leq b + t$$

Case 4>

For region $\quad \rho \geq b + t$

We use path L_4,

$$\oint_{L_4} H.dl = I_{enclosed} = I - I = 0$$

Or

$$H_{\Phi} = 0 \; A/m \qquad \rho \geq b + t$$

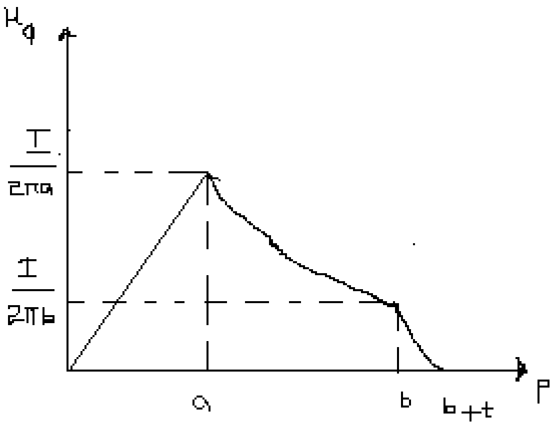
$$H = \frac{I\rho}{2\pi a^2} i_{\Phi} \qquad 0 \leq \rho \leq a$$

$$\frac{I}{2\pi\rho} i_{\Phi} \qquad a \leq \rho \leq b$$

$$\frac{I\left(1-\frac{[p^2-b^2]}{(t^2+2bt)}\right)}{2\pi\rho}\; i_\Phi \quad b\le\rho$$

$$0 \qquad \rho \ge b+t$$

→ The magnitude of 'H ' is sketched as follows



Example:

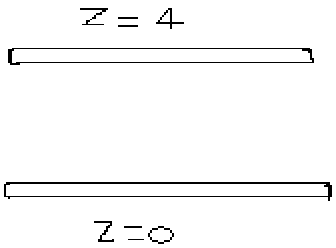
Planes z=0 and z=4 ccarry current $K=-10\,i_x$ and $k=10\,i_x\,{}^{A}/_{m}$ respectively.Determine H at (a) (1,1,1) & (b) (0,-3,10).

Solution:

Let the parallel sheets are as shown in figure

$$Z=0 \rightarrow k = -i_x\,{}^{A}/_{m}$$

$$Z=4 \rightarrow k = +i_x\,{}^{A}/_{m}$$



H at (1,1,1)

$0 < z = 1 < 4$

$$H = \frac{1}{2}\ k \times i_N.$$

$$H = H_0 + H_4$$

$$H_0 = \frac{1}{2}(-10 i_x \times i_z)$$

$$H_0 = -5(-i_y) = 5 i_y\, A/m$$

$$H_4 = \frac{1}{2}(10 i_x \times -i_z) = 5 i_y\, A/m$$

$$H = H_0 + H_4 = 10 i_y\, A/m.$$

(b)> H at (0,-3,10)

$Z = 10 > 4 > 0$

$H_0 = \frac{1}{2}(-10i_x \times i_z) = 5i_y \, {}^{A}/_{m}$

$H_4 = \frac{1}{2}(10i_x \times i_z) = -5i_y \, {}^{A}/_{m}$

Hence

$H = H_0 + H_4 = 0 \, {}^{A}/_{m}.$

Example;

A conducting plane at y = 1 carries a surface current of $10i_z \, m \, {}^{A}/_{m}$. Find H at

(0,0,0) and at (2,2,2).

Solution;

Given y=1 plane

$K = +10i_z \times 10^{-3} \, {}^{A}/_{m}$

$\rightarrow H \quad at \, (0,0,0)$. (2,2,2) y=2

$Y = 1 \, plane$

.(0,0,0) y=0

$y = 0 < y = 1$

$H = \frac{1}{2} k \times (-i_N)$

$$H = \frac{1}{2}(10i_z \times -i_y)\ m\ {}^{A}/_{m}$$

$$H = -5(-i_x)\ m\ {}^{A}/_{m}$$

$$H = 5(i_x)\ m\ {}^{A}/_{m}$$

→H at (0,0,0)

$y = 2 > y = 1\ plane$

i,e; the point is above the sheet of current

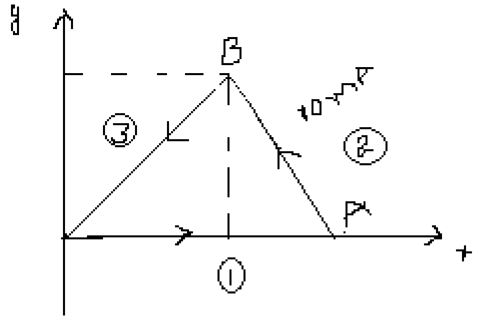
$$H = \frac{1}{2} k \times (+i_N)$$

$$H = \frac{1}{2}(10i_z \times i_y)\ m\ {}^{A}/_{m}$$

$$H = -5(i_x)\ m\ {}^{A}/_{m}.$$

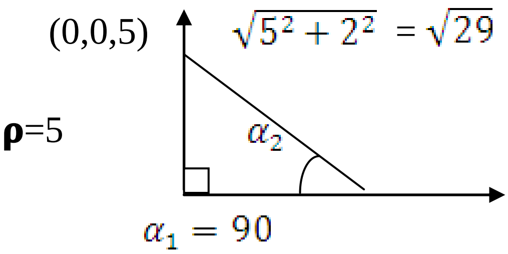
$Example$:

Find $\overrightarrow{H}$ and $\overrightarrow{B}$ at (0,0,5) due to side OA and side OB of the triangular current loop shown in fig. below.



Solution:

To find H at (0,0,5) due to side OA of the loop the fig. shown below



→ side 1 is treated as a straight conductor.We join the point of interest (0,0,5) to the beginning and end of the line current.

$$\cos\alpha_1 = \cos 90^0 = 0$$

$$\cos\alpha_2 = \frac{2}{\sqrt{29}} \ , \rho = 5$$

$$i_\Phi = i_x \times i_z = -i_y$$

→ The magnitc field intensity due to a finite straight conductor is given by

$$H = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)i_\Phi$$

$$H = \frac{10mA}{4\pi \times 5}(\frac{2}{\sqrt{29}} - 0)(-i_y)$$

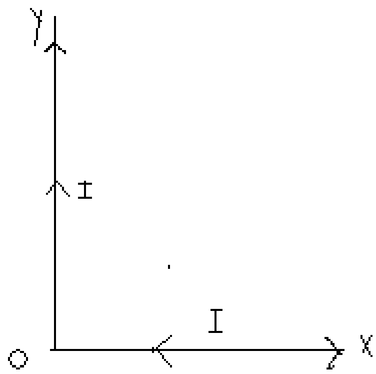
$$H = -\frac{1}{\sqrt{29}\pi} i_y \ {}^{m\,A}/_{m}.$$

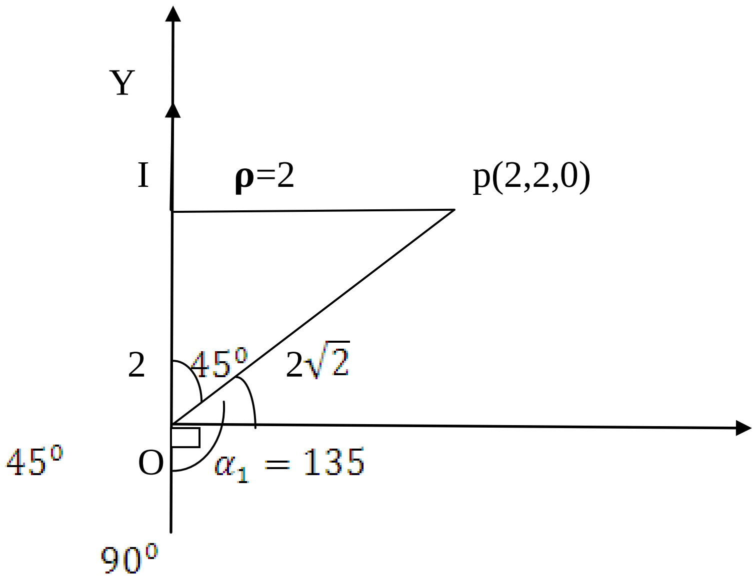
$$H = -0.0591 \ i_y \ {}^{m\,A}/_{m}$$

$$H = -59.1 \ i_y {}^{\mu\ A}/_{m}.$$

Example:

An infinitely long conductor is bent in an 'L' shape as shown in fig. below. If I= 5 find the field and flux densities at (2,2,0) (0,-2,0) and (0,0,2).





→ The expression for magnetic field intensity at any point p due to a straight filamentary conductor of finite length is

$$H = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)i_\Phi$$

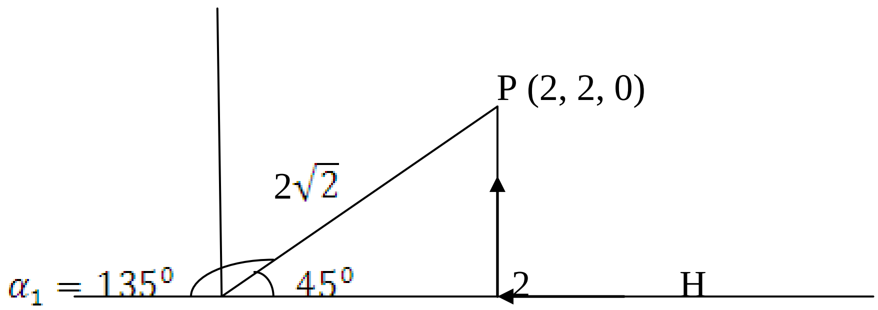
→ due ti current 'I' flowing through y-axis

$$\alpha_2 = 0^0 \quad \alpha_1 = 135^0$$

$$H_1 = \frac{5\times10^{-3}}{4\pi\times2}(\cos o^0 - \cos 135^0)\times -i_z$$

$$H_1 = \frac{5\times10^{-3}}{8\pi}(1-(-0.707))\times -i_z$$

$$H_1 = -\ \frac{5\times10^{-3}}{8\pi}(1.707)i_z$$



$$H_2 = \frac{5\times10^{-3}}{4\pi\times2}(\cos o^0 - \cos 135^0)(-i_x\times i_y)$$

$$H_2 = -\ \frac{5\times10^{-3}}{8\pi}(1.707)i_z$$

Hence

$$H\ =\ H_1 + H_2$$

$$H\ = -\ \frac{2\times5\times10^{-3}}{8\pi}(1.707)i_z$$

$$H = -0.6792 i_z \; {}^{m\,A}/_{m}$$

$$\rightarrow \; (0,2,0) \;\; H_1 = 0 \quad H_2 = \frac{I}{8\pi}\, i_z$$

$$H_2 = 0.1989\, i_z$$

$$\rightarrow \; (0,0,2)\; H_1 = \frac{I}{8\pi}\, i_x$$

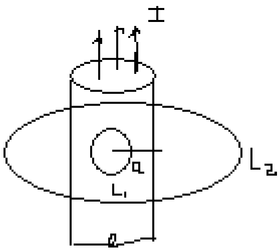
$$H_2 = \frac{I}{8\pi} i_z$$

$$H = H_1 + H_2 = 0.1989\, i_x \; {}^{m\,A}/_{m} + 0.1989 \; {}^{i_y m\,A}/_{m}$$

Example;

An infinitely long straight conducting rod of radius ‘a ‘ carries a current of ‘I’ in Z direction. Using Ampere’s circuital law, find H in all regions and sketch the variation of as a function of radial distance. If I= 3mA, and a = 2cm . find H and B at (0.1cm,0) and (0,4,0).

Solution;



From Ampere's law

$$\oint_L H.dl = I_{enclosed}$$

$$0 \le p \le a$$

$$\oint_{L_1} H.dl = I_{enclosed}$$

Consider $\oint_{L_1} H.dl = \oint H_\Phi d\Phi . \rho d\Phi i_\Phi$

$$\rho H_\Phi \int_{\Phi=0}^{2\pi} d\Phi \quad = \quad 2\pi \rho H_\Phi$$

Consider

$$I_{enclosed} = \int_s J.ds$$

$$J = \frac{I}{\pi a^2} i_z$$

$$I_{enclosed} = \int_s \frac{I}{\pi a^2} i_z . \rho d\rho d\Phi i_z$$

$$I_{enclosed} = \frac{I}{\pi a^2}\int_0^p \rho \, d\Phi . \int_0^{2\pi} d\Phi = \frac{Ip^2}{a^2}$$

Therefore

$$2\pi \, \rho H_\Phi = \frac{Ip^2}{a^2}$$

$$H_\Phi = \frac{I\rho}{2\pi a^2} i_\Phi \; {}^{A}/_{m} \quad 0 \leq p \leq a$$

$\rightarrow if \; a \leq \boldsymbol{\rho}$

By applying Ampere's law to path L2, we get

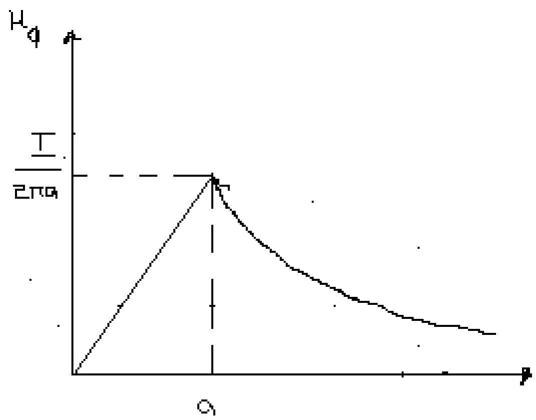
$$\oint_{L_2} H.dl = I_{enclosed}$$

$$2\pi\rho H_\Phi = I$$

$$H_\Phi = \frac{I}{2\pi\rho}$$

$$H = \frac{I}{2\pi\rho} i_\Phi \; {}^{A}/_{m} \quad a \leq \boldsymbol{\rho}$$

The magnitude of H is sketched as follows



$\rightarrow I = 3cm \qquad a = 2cm$

$$H\ at\ (0,4,0)$$

$$\rho = \sqrt{0 + 4^2}$$

$\rho > a$

$$H = \frac{I}{2\pi\rho} = \frac{3}{2\pi 4} = \frac{3}{8\pi} A/m$$

H at (0,1,0)

$$\rho = \sqrt{0 + 1^2} = 1$$

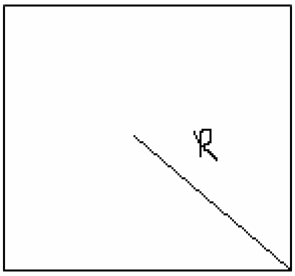
$\rho < a$

$$\therefore H_\Phi = \frac{I\rho}{2\pi a^2} = \frac{15}{4\pi} A/m$$

Ex: A square conducting loop of side a carries a current I in clockwise direction and is in z=0 plane. Find the field and the flux density at the center of the loop.

Sol

L/2



Choose a Cartesian coordinate system such that the loop is located as shown in fig below.

By symmetry, each half side contributes the same amount to H at the center.

For half side 0≤x≤L/2, y= -L/2, the biot- savarts law gives for the field at the origin.

$$dH = \frac{\mathrm{Idx}i_x * (-\mathrm{x}i_x + (\mathrm{L}/2)\ i_y)}{4\pi[x^2 + (L/2)^2]^{3/2}}$$

$$dH = \frac{Idx(\frac{L}{2})i_z}{4\pi[x^2 + (L/2)^2]^{3/2}}$$

Therefore the total field at the origin is

$$H = 8 \int_{0}^{L/2} \frac{I dx (\frac{L}{2}) i_z}{4\pi [x^2 + (L/2)^2]^{3/2}}$$

$$Let \;\; x = (L/2) \; tan\theta$$

$$dx \;\; = \;\; (L/2) * (sec\,\theta)^2 d\,\theta$$

$$H = 8 * \int_0^{45} [\frac{I(L/2)^2 (\sec\theta)^2 \mathrm{d}\,\theta}{4\pi (\frac{L}{2})^3 * (\sec\theta)^3}] i_z$$

$$H = 8 * \int_0^{45} \frac{I \cos\theta}{4\pi (\frac{L}{2})} * i_z$$

$$H = \frac{4I}{L\pi} [\sin\theta]_0^{45} \; i_z$$

$$H = (\frac{4I}{L\pi}) * (\frac{1}{\sqrt{2}}) i_z$$

$$H = \frac{2\sqrt{2}}{L\pi} I \; i_z \;\;\; A/m$$

MAGNETIC FLUX (φ)

A Magnetic field can be visualized as comprising of lines of force. These lines of force collectively referred as magnetic flux. Magnetic flux is denoted by φ Magnetic flux should be measured in Weber's.

MAGNETIC FLUX DENSITY (B)

Magnetic flux density is defined as the magnetic flux crossing from unit normal surface area and is denoted by B.

Magnetic flux density is a vector

The units for magnetic flux density is Weber's/square meter or Tesla.

$$B = \varphi/S$$

The magnetic flux through a surface S is given by

$$\Phi = \int_s B.ds$$

An isolated magnetic charge does not exist.

Thus the total flux through a closed surface in a magnetic field must be zero

$$\oint_s B.ds = 0$$

The total flux leaving a surface is the same as the total flux returning to the surface and Type equation here. The net flux is zero.

$$\oint_s B.ds = 0$$

By applying the divergence theorem to the above equation

$$\oint_s B.ds = \oint_v (\nabla . \mathrm{B})\, \mathrm{dv} = 0$$

$$\nabla . B = 0$$

This is the fourth Maxwell's equation

Hence the divergence of flux density in relation to the steady magnetic field is zero and is the gauss law for magnetic field.

Therefore the magnetic fields have no sources or sinks. Magnetic field lines are always continuous

The magnetic flux line is the path to which B is tangential at every point in the magnetic field.

The Magnetic flux density is related to the magnetic field intensity 'H' by

$$B = \mu_0 H$$

Where is a μ_0 constant known as the permeability of free space.

The units of permeability is Henry/meter and the value of μ_0 is $4\pi\times10^{-7}$ H/m

For any other medium

$B = \mu_r \mu_0 H$

μ_r = Relative permeability of free space

For air or free space μ_r is 1.

The flux density at any point p is given by $dB = \mu\, dH$

$$dB = \frac{\mu}{4\pi * R^2} * Idl * i_R \ wb/m^2$$

Expression for $\vec{H}$ at any point due to an ideal solenoid of infinite length

A solenoid is a cylindrically shaped coil consisting of large number of closely spaced turns.

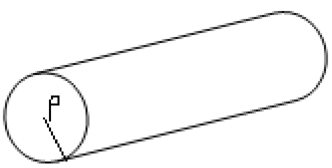
Solenoids are useful for storing magnetic energy.

Consider an infinitely long solenoid of radius 'a' and uniform current density $k_a i_\phi$ A/m as shown in fig a.

Using the concept of field due to two infinite parallel sheets we get the field

$H = k_a i_z \ for \ \rho < a$

$H = 0 \quad for \ \rho > a$



$K = k_a i_\varphi$

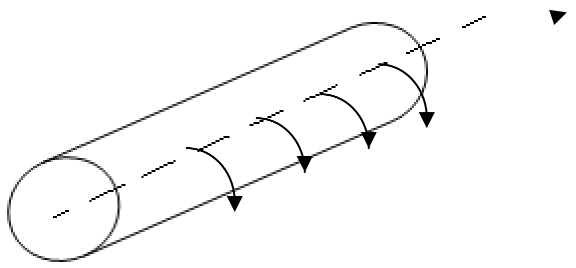
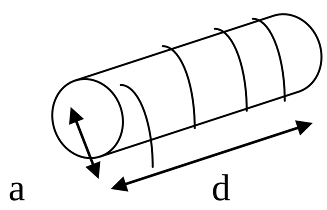


Fig a

If the solenoid has a finite length ‘d’ and consists of N closely turns , ‘a’ is the radii of solenoid and carries a current I.



N turns

If the turns are placed very close to each other so that it will form a surface current density

$$K = \frac{NI}{d} \; i_\varphi \; A/m$$

The magnetic field intensity inside the solenoid is given by

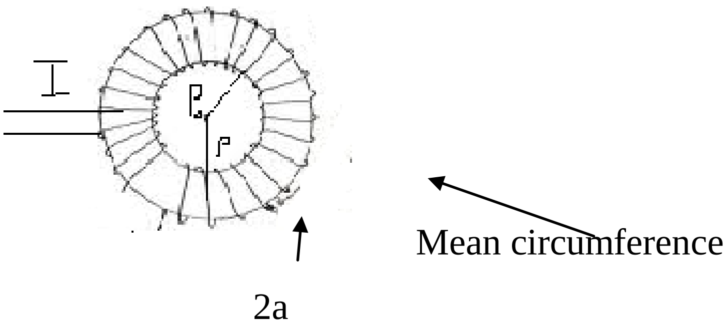
$$H = \frac{NI}{d} i_z \, A/m$$

Where N = Number of turns

d = length of the solenoid and I = solenoid current

Expression for $\vec{\boldsymbol{H}}$ at any point due to ideal toroid

If a long solenoid is bent in the form of a ring, thereby closed on itself it becomes a toroid.



a = width of the toroid

Toroid of N turns

When current I is passed through an exciting coil wound round the toroid , magnetic flux set up.

The net current enclosed by the toroid is from ampere's law.

$$\oint_L H . dl = I_{enc}$$

The mean circumference of the ring may be taken as the length of the magnetic circuit.

$$\oint dl = 2\pi\rho \ \ where\ p\ means\ radius.$$

$$\oint H.dl = NI$$

$$2\pi\rho\ H_\varphi = NI$$

$$H_\varphi = \frac{NI}{2\pi\rho}$$

Outside the toroid $H = 0$

The ideal toroid

$$H = k_a \left(\frac{\rho_0 - a}{\rho}\right) i_\varphi \ \ inside\ toroid$$

$$H = 0\ outside$$

$$K = k_a i_z\ \ at\ \ \rho = \rho_0 - a\ , z = 0$$

Ex: If $\vec{H}$ = yi_x-xi_y A/m on Z=0 plane , determine the current density . Define the relations used.

Sol: Given $\vec{H}$ = yi_x-xi_y A/m

$Z = 0\ plane$

$J = ?$

From amperes circuital law we have

$$\oint_L H.dl = I_{enc}$$

Or

$$\nabla \times H = J$$

$$J = \nabla \times H = \begin{vmatrix} i_x & i_y & i_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & -x & 0 \end{vmatrix}$$

$$J = -2\ i_z\ A/m^2$$

$\nabla \times H = J$ is called the Amperes law in differential form or Maxwell's third equation for magneto static.

Ex: In a conducting medium H=$y^2 Z i_x + 2(x+1)yz\, i_y - (x+1)z^2 i_z$. Find the current density at (1, 0,-3) and calculate the current passing through y=1 plane given 0≤x≤1.

$$J = \nabla \times H$$

$$\begin{vmatrix} i_x & i_y & i_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 z & 2(x+1)yz & -(x+1)z^2 \end{vmatrix}$$

$$= \; i_x \left[-\frac{\partial}{\partial y}((x+1)z^2) \; - \; \frac{\partial}{\partial z}(2(x+1)yz) \right]$$

$$-i_y \left[-\frac{\partial}{\partial x}((x+1)\, z^2) \; - \; \frac{\partial}{\partial z}(y^2 z) \right]$$

$$+ \; i_z \left[\frac{\partial}{\partial x}(2(x+1)yz) - \frac{\partial}{\partial y}(y^2 z) \right]$$

$$J \; = \; -2(x+1)\, y\, i_x \; + \; (y^2 \; + \; z^2)\, i_y$$

Current density at (1, 0,-3) is

$$J \; = \; -2(x+1)\, y\, i_x \; + \; (y^2 \; + \; z^2)\, i_y$$

$$= \; -2(1+1) * 0 \; i_x \; + \; (0+9)\, i_y$$

$$J \; = \; 9 i_y$$

Current $I = \int_s J.ds$

Given y=1 plane

$0 \leq x \leq 1, 0 \leq z \leq 1$

$$I = \int_s [-2(x+1)yi_x + (y^2 + z^2)i_y]dxdy$$

$$I = \int_s (y^2 + z^2)dxdz$$

$$I = \int_{x=0}^{1} dx \int_{z=0}^{1} (y^2 + z^2)\, dz$$

$$I = 1 * [1 + 1/3] = 4/3$$

$$I = \frac{4}{3}\, amp$$

Ex: Determine the magnetic flux for the surface described by

a)$\rho = 1m\ ,\ 0 \leq \varphi \leq \pi/2\ \ , 0 \leq z \leq 2m$

B) A sphere of radius 2m, if the magnetic field is of the form $H = \frac{1}{p} \cos\varphi \, i_\rho \; A/m$

Sol: $H = \frac{1}{p} \cos\varphi \, i_\rho \; A/m$

$B = \mu H$

$$B = \frac{\mu}{\rho} \cos\varphi \; i_\rho$$

$$\varphi = \int_s B.ds$$

$$\varphi = \int_s \left(\frac{\mu}{\rho} \cos\varphi \; i_\rho\right) * (\rho \, d\varphi \, dz \; i_\rho)$$

$$\varphi = \int_s \frac{\mu}{\rho} \cos\varphi \; d\varphi dz$$

$$\varphi = \frac{\mu}{\rho} \int_{\varphi=0}^{\pi/2} \frac{\mu}{\rho} \cos\varphi \int_{z=0}^{2} dz$$

$$\varphi = \frac{\mu}{\rho} * 1 * 2$$

$$\varphi = 2\mu$$

$$\varphi = 2\mu_0 = 8\pi * 10^{-7} webers.$$

MAGNETIC SCALAR POTENTIAL

In Electrostatics the scalar potential is given by

$$E = -\nabla v \qquad 1$$

By analogy with electrostatics similar relationship is oriented between magnetic field intensity 'H' and magnetic scalar potential V_m

$$H = -\nabla V_m \text{ provided } J = 0 \qquad 2$$

We _know that maxwell's third equation (or) the Ampere's circuit law in differential form is given by

$$\nabla \times H = J \qquad 3$$

By substituting equation 2 in eqn 3

$$\nabla \times (-\nabla V\_m) = J$$

$$J = 0$$

i.e. scalar magnetic potential can exist in a region where no current is exist (or) current density must be zero the scalar magnetic potential V_m is applied to fields due to permanent magnets

Also we know that

$\nabla . B = 0$ $\qquad\qquad\qquad B = \mu H$

$\nabla . (-\nabla V\_m) = 0$

∇^2 $V\_m = 0$, $J = 0$

V_m continues to satisfy laplace's eqn in homogenious magnetic material,it is not defined to any region where the current density exists.

When no current is enclosed we may write for magnetic field

$$\oint_L H . dl = I = 0$$

Between any two points along the path in the field

$$\int_1^2 H . dl = V_{m1} - V_{m2}$$

MAGNETIC VECTOR POTENTIAL

H = -∇V_m where V_m is scalar magnetic potential . but this holds the good for such paths in which

$$\oint_L H.dl = 0$$

In magnetics,the source for producing magnetic field is a "current element". So it is reasonable to think of a potential which depends on current element.

Magnetic vector potential is denoted as "A"

The magnetic flux density'B' can be expressed as space derrivative of A,whether divergence or curl.divergence gives a scalar quantity ,curl which gives a vector quantity

$$B = curl\, A$$

$$B = \nabla \times A$$

Where 'A' is called magnetic vector potential the units for 'A' is web/meter

We know that $\nabla \times H = J$

$$\nabla \times \frac{B}{\mu} = J$$

$$\nabla \times B = \mu J$$

$$\nabla \times (\nabla \times A) = \mu J$$

$$\nabla(\nabla . A) - \nabla^2 A = \mu J$$

For the situation of study current we take $\nabla . A = 0$

Then $$-\nabla^2 A = \mu J$$

$$\boxed{\nabla^2 A = -\mu J}$$

The eqn is of the same form as poisson's eqn in electrostatics

$$\nabla^2 V = -\frac{\rho_v}{\epsilon}$$

By comparing both the eqns μ is analogous to $\frac{1}{\epsilon}$

J is analogous to ρ_v

In electrostatics potential due to the line charge

$$V = \int_l \frac{\rho_l dl}{4\pi \epsilon_0 R}$$

For the three standard current configure the magnetic vector potential 'A' is expressed as follows

For current element $A = \int_l \frac{\mu I dl}{4\pi R}$

For sheet current $A = \int_S \frac{\mu k ds}{4\pi R}$

For volume current $A = \int_V \frac{\mu J dv}{4\pi R}$

Alternate approach to obtain the expression for 'A' for current distribution:

Magnetic flux density at any point due to the current carrying conductor from Biot-Savart's law is given by

$$B = \frac{\mu_0}{4\pi}\int_l \frac{I\,dl' \times R}{R^3} \qquad 1$$

Where $\vec{R}$ is the distance vector from the line element dl' at the source point(x', y', z') to the field point (x, y ,z)

$$|\vec{R}| = [(x-x')^2 + (y-y')^2 + (z-z')^2]^{\frac{1}{2}}$$

$$\vec{R} = (x-x')i_x + (y-y')i_y + (z-z')i_z$$

Consider $\nabla(\frac{1}{R})$

$$\nabla(\frac{1}{R}) = -\frac{(x-x')i_x + (y-y')i_y + (z-z')i_z}{[(x-x')^2 + (y-y')^2 + (z-z')^2]}$$

$$\nabla(\frac{1}{R}) = -\frac{\vec{R}}{R^3} = -\frac{i_R}{R^2}$$

$$\frac{\vec{R}}{R^3} = -\nabla(\frac{1}{R}) \qquad 2$$

Where the differentiation is with respect to (x, y, z)

By substituting eqn (2) in eqn (1)

$$B = -\frac{\mu}{4\pi}\int_l I\,dl' \times \nabla\left(\frac{1}{R}\right) \longrightarrow \qquad 3$$

By applying the vector identity

$$\nabla \times (fF) = f\nabla \times F + (\nabla f) \times F$$

Where f is a scalar field and F is a vector field

Taking $f = \frac{1}{R}$ and $F = dl'$ we have

$$\nabla \times (\frac{1}{R} dl') = \frac{1}{R} \nabla \times dl' + (\nabla(\frac{1}{R})) \times dl')$$

Since the ∇ operates with respect to (x,y,z) while dl' as a function of (x', y', z')

$$\therefore \nabla \times dl' = 0$$

$$\therefore \nabla \times (\frac{1}{R} dl') = (\nabla(\frac{1}{R})) \times dl')$$

$$dl' \times \nabla\left(\frac{1}{R}\right) = -\nabla \times \left(\frac{1}{R} dl'\right) \longrightarrow \quad 4$$

Substituting eqn (4) in eqn(3) we have

$$B = \frac{-\mu I}{4\pi} \int_l -\nabla \times \frac{1}{R} dl'$$

$$B = -\int_l \nabla \times \frac{\mu I dl'}{4\pi R} \qquad (5)$$

$$B = -\nabla \int_l \times \frac{\mu I dl'}{4\pi R}$$

By comparing above eqn with

$$B = \nabla \times A$$

$$A = \int_l \frac{\mu I dl'}{4\pi R} \, l$$

We have

$$\Psi = \int_s B.ds$$

By substituting $B = \nabla \times A$ in above equation

$$\Psi = \int_s B.ds = \int_s \nabla \times A.ds$$

By applying strokes theorem in reverse direction

$$\Psi = \int_s B.ds = \int_s \nabla \times A.ds = \oint_L A.dl$$

$$\Psi = \oint_L A.dl$$

EX: Determine the magnetic flux, for the surface described by

i) $\rho = 1m, 0 \leq \emptyset \leq \frac{\pi}{2}, 0 \leq Z \leq 1$.

ii)a sphere of radius 2m,if the magnetic field is of the form $[H = \frac{1}{\rho} cos\ \emptyset] i_\rho\ AM$

SOL:Given $[H = \frac{1}{\rho} cos\ \emptyset] i_\rho$

i) $\emptyset = \int_s B.ds$ $\quad B = \mu H$

$$\emptyset = \int_s \frac{\mu}{\rho} cos\ \emptyset i_\rho . \rho d\emptyset\ dz i_\rho$$

$$\emptyset = \int_s \frac{\mu}{\rho} cos\ \emptyset \rho d\emptyset\ dz$$

$$\emptyset = \frac{\mu}{\rho} \int_{\emptyset=0}^{\frac{\pi}{2}} cos\ \emptyset\ d\emptyset \int_{x=0}^{2} dz$$

$$\emptyset = \frac{\mu}{\rho} [sin\emptyset]_{\emptyset=0}^{\frac{\pi}{2}} [z]_0^2$$

$$\emptyset = \frac{\mu \times 1.2}{\rho} = 2\mu\, Web$$

$$\emptyset = 2 \times 4\pi \times 10^{-7}\ webers\ in\ free\ space$$

ii $\quad [H = \frac{1}{\rho} cos\ \emptyset] i_\rho$

$$H = [\frac{1}{\rho} cos^2 \emptyset]\ i_x + [\frac{1}{\rho} cos\ \emptyset sin\emptyset] i_y$$

$$H = \left(\frac{1}{\rho} cos^2 \emptyset \, sin\theta \, cos \emptyset + \frac{1}{\rho} cos \emptyset \, sin^2 \emptyset \, sin\theta\right) i_r +$$

$$\left(\frac{1}{\rho} cos^2 \emptyset \, cos\theta \, cos \emptyset + \frac{1}{\rho} cos \emptyset \, sin^2 \emptyset \, cos\theta\right) i_\theta$$

$$H = \frac{1}{\rho} \; sin\theta \; cos\emptyset \; i_r + \frac{1}{\rho} \; cos\theta \; cos\emptyset \; i_\theta$$

$$\rho = \sqrt{x^2 + y^2} \qquad x = r \, sin\theta \, cos\emptyset$$

$$Y = r \, sin\theta \, sin\emptyset$$

$$\rho = \sqrt{r^2 \, sin^2 \, \theta} \, (cos^2 \emptyset + sin^2 \emptyset)$$

$$\rho = r \, sin\theta$$

$$H = \frac{1}{r} cos \emptyset \, i_r + \frac{1}{r} cot\theta \, cos \emptyset \, i_\theta$$

$$\emptyset = \int_s B.ds$$

$$\emptyset = \int_s \left[\frac{\mu}{r} cos \emptyset \, i_r + \frac{\mu}{r} cot\theta \, cos \emptyset \, i_\theta\right]. \; r^2 \, sin\theta \; d\theta \; d\emptyset$$

$$\emptyset = \int_s \frac{\mu}{r} \, r^2 \, cos\emptyset sin\theta \; d\theta \; d\emptyset$$

$$\emptyset = \mu r \int_{\theta=0}^{\pi} sin\theta \; d\theta \, . \int_{\emptyset=0}^{2\pi} cos\emptyset \; d\emptyset$$

$$\emptyset = \mu r[-cos\theta]_0^\pi \, [sin\emptyset]_0^{2\pi}$$

EX:For the magnetic field given by A = $x^2yi_x + y^2xi_y - 2xyzi_z$ wb/m, evaluate the magnetic field intensity at(1,2,3)and the resulting magnetic flux through the surface given z=1,0$\leq x \leq 1, -1 \leq y \leq 1$

Sol: Given $A = x^2yi_x + y^2xi_y - 2xyzi_z$

$$B = \nabla \times A = \begin{vmatrix} i_x & i_y & i_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & y^2x & 2xyz \end{vmatrix}$$

$$B = i_x[-\frac{\partial}{\partial y}(2xyz) - \frac{\partial}{\partial z}(y^2x)] - i_y\left[-\frac{\partial}{\partial x}(2xyz) - \frac{\partial}{\partial z}(x^2y)\right] + i_z[\frac{\partial}{\partial x}(y^2x) - \frac{\partial}{\partial y}(x^2y)]$$

$$B = -2xzi_x + 2yzi_y + (y^2 - x^2)i_z$$

$B\ at(1,2,3)$

$$B = -6i_x + 12i_y + 3i_z$$

$$H = \frac{-6i_x + 12i_y + 3i_z}{4\pi \times 10^{-7}}\ A/m$$

$$\emptyset = \int_s B.ds = \int_s (-2xzi_x + 2yzi_y + (y^2 - x^2)i_z).dxdy$$

$$= \int_s (y^2 - x^2)\, dxdy$$

$$= \frac{y^3}{3}]_{-1}^{1} x]_0^1 - \frac{x^3}{3}]_{-1}^{1} y]_{-1}^{1} = 0.$$

Ex: Obtain the vector magnitude potential due to a long straight conducting wire carrying a current ‘I’ in +z direction.

Sol: we know that magnetic field intensity due to a straight conducting wire carrying a current in +z direction is given by

$$B = \frac{\mu I}{2\pi\rho} i_\emptyset$$

Relation between B and magnetic vector potential ‘A’ is given by

$$B = \nabla \times A$$

$$\nabla \times A = B = \frac{\mu I}{2\pi\rho} i_\emptyset$$

Expanding $\nabla \times A$ in cylindrical co-ordinate system

$$\nabla \times A = \frac{1}{\rho}\begin{vmatrix} i_\rho & \rho i_\emptyset & i_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \emptyset} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\emptyset & A_z \end{vmatrix}$$

$$= \frac{1}{\rho}\left(\frac{\partial A_z}{\partial \rho} - \frac{\partial \rho A_\emptyset}{\partial \emptyset}\right) i_\rho - \frac{1}{\rho}\rho i_\emptyset\left(\frac{\partial A_z}{\partial \rho} - \frac{\partial A_\rho}{\partial z}\right) + \frac{1}{\rho} i_z\left(\frac{\partial \rho A_\emptyset}{\partial \rho} - \frac{\partial A_\rho}{\partial \emptyset}\right)$$

$$\nabla \times A = \left[\frac{1}{\rho}\frac{\partial A_z}{\partial \rho} - \frac{\partial A_\emptyset}{\partial \emptyset}\right] i_\rho + \left[\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right] i_\emptyset + i_z\left[\frac{\partial A_\emptyset}{\partial \rho} - \frac{1}{\rho}\frac{\partial A_\rho}{\partial \emptyset}\right]$$

Equating $\emptyset$ component of the cylindrical

$$\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho} = \frac{\mu I}{2\pi\rho}$$

It is evident hat ‘A’ cannot be function of ‘z’, since the filament is uniform with z

$$-\frac{\partial A_z}{\partial \rho} = \frac{\mu I}{2\pi\rho}$$

Taking integration w. r. t ‘ρ’ on both sides

$$+A_z = -\frac{\mu I}{2\pi} \, ln\rho + c$$

Where ‘c’ is the constant of integration

By applying initial conditions of $A_z = 0$

With $\rho = \rho_0$

$$A_z = -\frac{\mu I}{2\pi} ln\rho + \frac{\mu I}{2\pi} ln\rho_0$$

$$A_z = \frac{\mu I}{2\pi} ln\frac{\rho}{\rho_0}$$

Hence the magnetic vector potential is given by

$$A = \frac{\mu I}{2\pi} \, ln(\frac{\rho}{\rho_0}) \, i_z$$

MAGNETIC FIELD ABOUT A LONG STRAIGHT W:

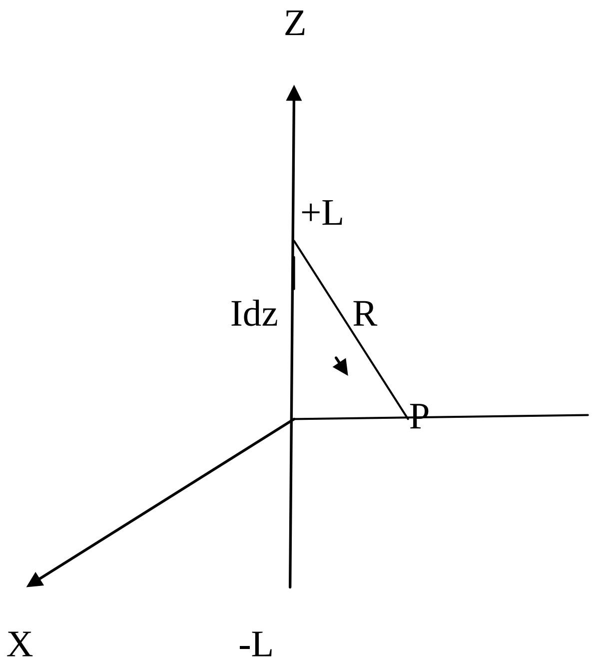
Using the vector potential let it be required to find the magnetic field strength about a long straight wire carrying a current

The general expression for vector potential is

$$A = \int_v \frac{\mu j dv}{4\pi R} = \int_L \frac{\mu j dl}{4\pi R} \longrightarrow \quad 1$$

The current is entirely in the Z-direction so that A has only one component A_z then in a homogeneous medium

$$A_z = \frac{\mu}{4\pi}\int_{-L}^{L} \frac{Idz}{R}$$



If the point 'P' is taken in the y-z plane

$$R = \sqrt{z^2 + y^2}$$

And

$$A_z = \frac{\mu}{4\pi}\int_{-L}^{L} \frac{Idz}{\sqrt{z^2+y^2}}$$

$$A_z = \frac{\mu}{2\pi}\int_{0}^{L} \frac{Idz}{\sqrt{z^2+y^2}}$$

$$= \frac{\mu I}{2\pi}[\ln [z + \sqrt{z^2 + y^2}]]_0^L$$

$$A_z = \frac{\mu I}{2\pi}[ln [L + \sqrt{L^2 + y^2}] - ln\, y]$$

$For\ L \gg y,$ the vector potential approximately by

$$A_z = \frac{\mu I}{2\pi}[\ln 2L - \ln y]$$

Then for a point in the y-z plane

$$H_x = \frac{1}{\mu}(\nabla \times A)_x$$

$$H_x = \frac{1}{\mu}\frac{\partial A_z}{\partial y}$$

$$H_x = -\frac{I}{2\pi y}$$

The lines of magnetic field strength will be circles about wire, tha t is in the $\emptyset$ direction for any arbitrary point p, not necessary in the y-z plane

$$H_\emptyset = \frac{I}{2\pi r}$$

Where $r = \sqrt{x^2 + y^2}$ is the distance of the point p from the wire

EX: obtain the vector magnetic potential due to infinite sheet of current.

SOL: consider an infinite current sheet in the z = 0 planes and the sheet has uniform current density $k = K_y\, i_y\, a/m$.

Then the magnetic field intensity is given by

$$H = \frac{1}{2} k i_x \quad A/M \qquad Z > 0$$

$$H = -\frac{1}{2} k i_x \ A/M \qquad Z < 0$$

Then $$B = \frac{\mu}{2} k i_x \ Weber/mt^2 \qquad Z > 0$$

$$B = -\frac{\mu}{2} k i_x \; Weber/mt^2 \qquad Z < 0$$

$$\nabla \times A = B$$

By equating X-component of the rectangular

$$\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} = b = \frac{\mu k}{2}$$

'A' is independent of x and y

$$-\frac{\partial A_y}{\partial z} = \frac{\mu k}{2}$$

Integrating w.r.t 'Z' on both sides

$$A_y = -\frac{\mu k}{2} z + c$$

Applying initial values as $A_y = 0$ ans $z = z_0$

Then $c = \frac{\mu k}{2} z_0$

$$A_y = -\frac{\mu k}{2}(z - z_0)$$

Ex:a medium of $\epsilon = 5 \epsilon_0$ $\mu = 20\mu_0$ $\sigma = 0.2\, m\, s/m$, $E = 20\mu\, v/m$. find the conduction current density .if this current density exists in a cylindrical rod of 2cm diameter evaluate the current that can flow through the rod.

SOL: given $\sigma = 0.2\; 10^{-3}\; s/m$

$$E = 20 \times 10^{-6}\; v/m$$

$$J = ?$$

Conduction current density $J = \sigma E$

$$J = 0.2 \times 10^{-3} \times 20 \times 10^{-6}$$

$$J = 4 \times 10^{-9} \; A/m^2$$

Current $\quad I = \int_s J.ds$

$$I = \int_s 4 \times 10^{-9} \, i_z . \rho \, d\emptyset \, d\rho \, i_z$$

$$\rho \;\; 1cm$$

$$I = 4 \times 10^{-9} \times \frac{\pi \times d^2}{4} \qquad d = 2\rho$$

$$I = 4\pi \times 10^{-9} \times 10^{-4}$$

$$I = 4\pi \times 10^{-13} \qquad amp.$$

Q. Obtain the vector magnetic potential due to infinite sheet of current

Sol- consider an infinite current sheet in the $Z = 0$ plane and has a uniform current density$k = k_y \, i_y A/M$.

Then the magnitude field intensity is given by $H = 1/2 \, K \, i_x \;\; A/m \quad z > 0$

$$H = -1/2 \, K \, i_x \;\; A/m \qquad z < 0$$

Then $\quad B = \mu/2 \;\; K \, i_x \quad weber/m^2 \qquad z > 0$

$$B = -\mu/2 \;\; K \, i_x \quad weber/m^2 \qquad z < 0$$

$\nabla \times A = B$

By equating x-component of the rectangular

$$\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} = B = \mu k/2$$

A is independent of x and y

$$-\frac{\partial A_y}{\partial z} = \mu k/2$$

Integrating w.r.t z on both sides

$$A_y = -\frac{\mu k}{2} z + C$$

Applying initial conditions as $A_y = 0 \quad z = z_0$

Then $C = \frac{\mu k}{2} z_0$

$$A_y = -\frac{\mu k}{2}(z - z_0)$$

Q. In a medium of $\epsilon = 5\epsilon_0 \quad \mu = 2.5\mu_0$ and $\sigma = 0.2 \times 10^{-3} \frac{s}{m}$, E=20 μ V/m. find the conduction current density. if the current density exists in a cylindrical rod of 2 cm diameter .evaluate the current that can flow through the rod

Sol- given $\sigma = 0.2 \times 10^{-3} \frac{s}{m}$

$$E = 20 \ \mu V/m$$

$$J = ?$$

Conduction current density $J = \sigma E$

$$J = 0.2 \times 10^{-3} \times 20 \ \mu$$

$$J = 4 \times 10^{-9} \ A/m^2$$

Current $I = \int j . ds$

$$I = \int 4 \times 10^{-9}\, i_{z\,.}\ \rho\, d\emptyset\, d\rho\, dz$$

$$I = 4 \times 10^{-9} \times (\pi \times d^2/4)$$

$$I = 4\pi \times 10^{-13} \quad Amp$$

INDUCTANCE

→ A capacitor when connected to a source potential would store energy in the electric tie

Work must be done to build up a magnetic field by passing a current in an inductor hence the inductor stores energy.
The inductance L of a coil may be defined as the ratio of the total magnetic flux linkage to the current I through the inductor.

$$L = \frac{N\emptyset}{i} = \frac{\lambda}{i}$$

The units for inductance is Henry or Weber
If the current is increasing from 0 to I with the Voltage across the inductor equal to V then the source is supplying power equal to VI.

Energy supplied by the source in time $dt = VI\, dt$

If we neglect the resistance in any part of the circuit, so that no energy is dissipated as heat .then the energy must be stored in the inductor

$$dw = VI\, dt$$

if the current is increased to I in time t then

$$w = \int VI\ dt$$

$$w = \int LI \frac{di}{dt}\ .dt \qquad since\ V = L\frac{di}{dt}$$

$$w = \int LI\ di$$

$$W = L\frac{i^2}{2} = \frac{\lambda}{i}.\frac{i^2}{2} = \frac{\lambda}{i}.\frac{1}{2} = \frac{\lambda^2}{2L}$$

This may be compared with expression for energy stored in capacitor

$$w = \frac{1}{2}CV^2 = \frac{1}{2}\ QV = \frac{1}{2}\frac{Q^2}{C}$$

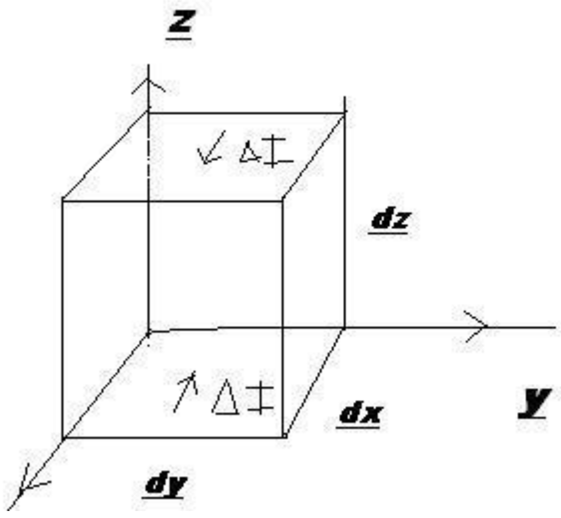
$Comparison$

$I = C.\frac{dV}{dt}$	$V = L\frac{di}{dt}$
$Q = \int i.dt$	$\lambda = \int V.dt$
$I = \frac{dQ}{dt}$	$V = \frac{d\lambda}{dt}$
$Q = CV$	$\lambda = LI$

$Energy\ density\ in\ magnetostatic\ fields$

→Consider a differential volume in a magnetic field as shown in the figure.

Let the volume be covered with the conducting sheets at the top and bottom surfaces with current Δi



→We assume that the volume is filled with such differential volume

The inductance of each volume is

$$\Delta L = \frac{\Delta\varphi}{\Delta i} = \mu H \Delta x \Delta z/\Delta i \longrightarrow \quad 1$$

Where $\Delta i = H\,\Delta y \longrightarrow \quad 2$

Substituting 2 in 1

$$\Delta L = \frac{\mu H \Delta x \Delta z}{H \Delta y} = \frac{\mu \Delta x \Delta z}{\Delta y} \quad 3 \longrightarrow$$

The energy stored in the inductor is

$$\Delta w = \frac{1}{2}\Delta L . \Delta i^2$$

$$\Delta w = \frac{1}{2}\frac{\mu \Delta x \Delta z}{\Delta y} . H^2 . \Delta y^2 = \frac{1}{2} . H^2 \mu . \Delta y\, \Delta x \Delta z$$

→The magneto static energy density is defined as

$$w_m = \lim_{\Delta v \to 0} \frac{\Delta w_m}{\Delta V}$$

$$w_m = \frac{1}{2} \mu H^2$$

Hence

$$w_m = \frac{1}{2} \mu H^2 = \frac{1}{2} B.H = \frac{B^2}{2\mu}.$$

Thus the energy in a magneto static field in a linear medium is

$$W_m = \int w_m . dv$$

$$W_m = \frac{1}{2} \int B.H.dv = \frac{1}{2} \int \mu H^2 . dv$$

Forces due to magnetic fields

There are three ways in which force due to magnetic fields can be experienced.

a→due to moving charged particle in B field

b→ on a current element in an external B field

c→ between two current elements.

A. Force on charged particle

→Force on stationary or moving charge Q in an electric field is given by

$$F_e = QE \qquad 1 \longrightarrow$$

If Q is positive charge force will have the same direction as that of E

→ A magnetic field can exert force only on moving charge.

This force magnitude is proportional to the magnitude of the charge Q, its velocity v and the flux density B and the sine of the angle b/w the vectors v and B.

$F_m = QvB \sin\theta$

$$F_m = Qv \times B \longrightarrow \quad 2$$

→ For a moving charge in the presence of both electric and magnetic fields, the total force on the charge is given by

$F = F_e + F_m$

$F = QE + Qv \times B$

$$F = Q[E + v \times B] \qquad \backslash$$

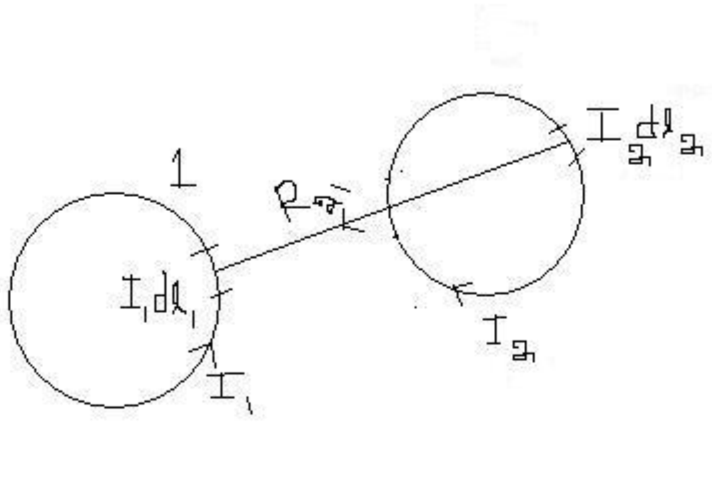
This is known as the Lorentz force equation

Force on a charged particle

State of particle	*E field*	*B field*	*Combination of E & B*
stationary	QE	$-$	QE
moving	QE	$Qv \times B$	$QE + Qv \times B$

C. Force between two current elements

Consider two loops designated c1 and c2 carry currents I_1 *and* I_2 as shown in the figure below



According to the biat-savart's law, both current elements produce magnetic fields.

The magnetic flux density produced by element $I_2\, dl_2$ is given by

$$dB_2 = \frac{\mu_0\, I_2\, dl_2 \times i_{R_{21}}}{4\pi R_{21}{}^2}$$

The force on $I_1\, dl_1$ due to magnetic flux dB_2 produced by element $I_2\, dl_2$ is given by

$$d(dF_1) = I_1\, dl_1 \times dB_2$$

$$d(dF_1) = \frac{\mu_0\, I_1\, dl_1 \times \,(I_2\, dl_2 \times i_{R_{21}})}{4\pi R_{21}{}^2}$$

We obtain the total force F_1 on current loop c1 due to current loop c2 is given by

$$F_1 = \frac{\mu_0\, I_1\; I_2}{4\pi} \oint_{C1} \oint_{C2} \frac{dl_1 \times (dl_2 \times i_{R_{21}})}{R_{21}{}^2}$$

Force F_2 on the loop 2 due to the magnetic flux produced by loop 1 is obtained as

$$F_2 = \frac{\mu_0\, I_1\; I_2}{4\pi} \oint_{C1} \oint_{C2} \frac{dl_2 \times (dl_1 \times i_{R_{12}})}{R_{12}{}^2}$$

Ex:- find the magnitude and nature of the resulting force between two long parallel straight filamentary conductors , oriented along z direction and separated by a distance of 1 cm , if they carry current of 5 mA 1. Same direction, 2. Opposite direction

$$F = I_1 i_z \times B_2$$

$$B_2 = -\frac{\mu_0 \, I_2 \, i_\emptyset}{2\pi d} \; weber/meter \; square$$

$$F = +\frac{\mu \quad I_1 \, I_2 \, i_\rho}{2\pi d} \quad N/M$$

2. The force of repulsion between two infinite long, straight, parallel, filamentary conductors with separation d and carrying opposite current I is given by

$$F = +\frac{\mu_0 I^2}{2\pi d} N/M \;\; (I_1 = I_2)$$

$$F = \frac{4\pi \times 10^{-7} \times 25 \times 10^{-6}}{2\pi \times 10^{-2}} N/M$$

$$F = 5 \times 10^{-10} \quad N/M$$

$nature\ of\ the\ force\ is\ repulsive$

1. If the current is in the same direction .the force is attractive and is given by

$F = -(\mu\_0\ I\text{^}2)/2\pi d \quad i\_\rho \;\; N/M$

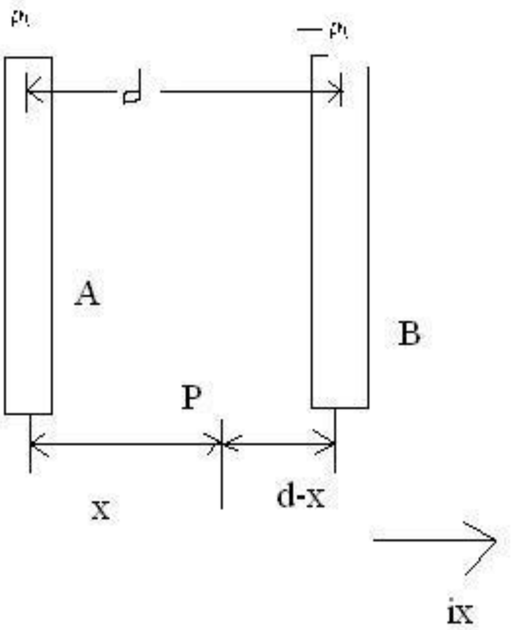
$$F = -5 \times 10^{-10} \quad i_\rho \quad \text{N/M}$$

Capacitance of parallel wires or two wire lines

It is given by $C = \frac{\pi\epsilon}{\ln\frac{d}{r}}$ F/M

d = distance between the wires

r= radius of each conducting wire.



Let ρ_l and $-\rho_l$ be the line charge densities of the lines A and B respectively.

Let d be the distance between the wires and r be the radius of each wire.

Electric field at p due to ρ_l is

$$E_1 = \frac{\rho_l}{2\pi\epsilon x}\ i_x$$

Electric field due to $-\rho_l$ at p is given by

$$E_2 = +\frac{\rho_l}{2\pi\epsilon(d-x)}\ i_x$$

The potential difference V is given by

$$V = -\int_{A}^{B} E.dx\, i_x = -\int_{d-r}^{r} E.dx\, i_x$$

$$V = -\int_{d-r}^{r} \frac{\rho_l}{2\pi\epsilon x}\, i_x.dx\, i_x - \int_{d-r}^{r} \frac{\rho_l}{2\pi\epsilon(d-x)}\, i_x.dx\, i_x$$

$$V = -\int_{d-r}^{r} \frac{\rho_l}{2\pi\epsilon x}\, .dx - \int_{d-r}^{r} \frac{\rho_l}{2\pi\epsilon(d-x)}\, .dx$$

$$V = -\frac{\rho_l}{2\pi\epsilon}[\int_{d-r}^{r} \frac{1}{x}\, .dx - \int_{d-r}^{r} \frac{1}{(d-x)}\, .dx\,]$$

$$V = -\frac{\rho_l}{2\pi\epsilon}\left[\ln(\frac{r}{d-r}) - \ln(\frac{r}{d-r})\right]$$

$$= \frac{\rho_l}{2\pi\epsilon} 2\ln\left(\frac{d-r}{r}\right)$$

In all practical cases d>>r .as a result we have $\left(\frac{d-r}{r}\right) \approx \frac{d}{r}$

$$V = \frac{\rho_l}{\pi\epsilon} ln\frac{d}{r}$$

The capacitance $C = \frac{\rho_l}{V}\ F/M$

$$C = \frac{\pi\epsilon}{\ln\frac{d}{r}} \quad F/M$$

Capacitance of pair of parallel wires of length L is given by

$$C = \frac{\pi \epsilon L}{\ln \frac{d}{r}} \quad Farad$$