Maxwell's equations for the time varying fields

Electrostatic fields are usually produced by the static electric charges where as magneto static fields are due to motion of electric charges with uniform velocity.

Time varying fields (or) waves are usually due to accelerated charges (or) time varying currents.

Stationary charges → Electro static fields

Steady currents → Magneto static fields

Time varying currents → Electromagnetic fields

Michael Faraday and Joseph Henry discovered that a time varying magnetic field would produce an electric current.

Faraday's Law

According to Faraday's experiments, a static magnetic field produces no current flow but a time varying field produces an induced voltage (called electromotive force) in a closed circuit which causes a flow of current. Faraday discovered that the induced emf V_{emf} (in volts) in any closed circuit is equal to the time rate of change of the magnetic flux linkage by the circuit .This is called "Faraday's Law".

And it can be expressed as

$$Vemf = -\frac{d\lambda}{dt}$$

$$Vemf = -\frac{Nd\varphi}{dt} \longrightarrow \quad 1$$

Where **N** is the number of turns in the circuit and **φ** is the flux through each turn. The negative sign shows that the induced voltage acts in such a way as to oppose the flux producing it. This is known as “Lenz’s law”. The direction of current flow in the circuit is such that the induced magnetic field produced by the induced current will oppose the original magnetic field.

For a circuit with a single turn (N=1), then equation (1) becomes

$$Vemf = -\frac{d\varphi}{dt} \longrightarrow \quad 2$$

In terms of E and B the above equation becomes

$$Vemf = \oint_L E.dl = -\frac{d}{dt}\int_s B.ds$$

Transformer and motional EMF’S

The variation of flux with time may be caused in three ways

1. By having a stationary loop in a time varying B - field.
2. By having a time varying loop area in a static B - field.
3. By having a time varying loop area in a time-varying B field.

Stationary Loop in Time varying B – field (Transformer EMF)

Time varying magnetic field produces an induced voltage in a closed loop is equal to the negative of the time rate of change of magnetic flux enclosed by the path.

$$Vemf = \oint_L E.dl = -\frac{d\varphi}{dt} = -\frac{d}{dt}\int_s B.ds$$

$$\oint_L E.dl = Vemf = -\int \frac{\partial B}{t}.ds \longrightarrow \quad 1$$

This **emf** induced the time varying magnetic field '**B**' in a stationary loop is often referred to as "**Transformer emf**".

By applying Stoke's Theorem to the equation (1)

$$\oint_L E.dl = \int_s (\nabla \times E).ds = -\int_s \frac{\partial B}{\partial t}.ds$$

Equating the surface integrals

$$\nabla \times E = -\partial B/\partial t$$

This is one of the Maxwell's equations for time varying fields. The time varying electric field is not conservative.

Moving loop in static B field (motional emf)

When a conducting loop is moving in a static magnetic field, an emf is induced in the loop. The force on a charge moving with uniform velocity **v** in a magnetic field is

$$Fm = Q\,\boldsymbol{v} \times B$$

We define motional electric field E_m as

$$Em = \frac{F_m}{Q} = \boldsymbol{\nu} \times B$$

If we consider a conducting loop, moving with uniform velocity **v** as consisting of a large number of free electrons, the emf induced in the loop is

$$Vemf = \oint_L E_m \,.\, dl = \oint_L (\boldsymbol{\nu} \times B) \,.\, dl$$

This type of emf is called “**motional emf**” and this kind of emf found in electrical machines such as motors, generators and alternators.

Moving loop in time – varying field

This is the general case in which a moving conducting loop is in a time varying magnetic field. Both transformer **emf** and motional **emf** are present.

$$Vemf = \oint_L E.\,dl = -\int_s \frac{\partial B}{\partial t} \,.\, ds + \oint_L (\mathbf{v} \times \mathrm{B}).\, dl$$

$$\nabla \times E = -\frac{\partial B}{\partial t} + \boldsymbol{\nabla} \times (\boldsymbol{\nu} \times B)$$

The equation of the continuity for time varying fields

Due to the principle of charge conservation, the time rate of decrease of charge within a given volume must be equal to the net outward flow through the closed surface of the volume.

The current $\mathbf{I}_{out}$ of the closed surface is

$$\mathbf{I}_{out} = \oint_s \boldsymbol{J}.ds = -\frac{d\,Q_{in}}{dt} \longrightarrow \quad 1$$

where $\mathbf{Q}_{in}$ is the total charge enclosed by the closed surface

$$\oint_s \boldsymbol{J}.ds = -\frac{d}{dt}\int_v \rho_v.\,dv$$

$$\because\ \mathbf{Q}_{in} = \int_v \rho_v.\,dv$$

$$\oint_s \boldsymbol{J}.ds = -\int_v \frac{\partial \rho_v}{\partial t}.\,dv \longrightarrow \quad 2$$

By applying divergence theorem to equation (2)

$$\int_v (\nabla.\mathrm{J}).\,dv = -\int_v \frac{\partial \rho_v}{\partial t}.\,dv$$

$$\nabla.\mathrm{J} = -\frac{\partial \rho_v}{\partial t}$$

This is the time varying form of the "equation of continuity".

Inconsistency of Ampere's law

Taking the divergence of ampere's law yields the equation of continuity for steady currents.

Ampere's law is $\nabla \times H = J \longrightarrow$ 1

Taking the divergence on both sides of equation 1 gives

$$\nabla.(\nabla \times H) = \nabla.J$$

but the divergence of curl of any vector is identically zero.

$$\therefore \nabla.J = 0 \longrightarrow 2$$

Thus the ampere's law is not consistent with the time varying equation of continuity

i.e $\therefore \nabla.J = -\frac{\partial \rho_v}{\partial t} \neq 0 \longrightarrow$ 3

This needs a modification to ampere's law.

The correct modification may be found by substituting gauss law in equation(3)

$$\nabla.J = -\frac{\partial \rho_v}{\partial t} = -\frac{\partial}{\partial t}(\nabla.D)$$

Interchanging the differentiation w.r.t space and time gives

$$\nabla.J = -\nabla.\frac{\partial D}{\partial t}$$

$$\nabla.\left(\frac{\partial D}{\partial t} + J\right) = 0 \longrightarrow 4$$

This may be put into integral form by integrating over a volume and then applying the divergence theorem.

$$\oint_s \left(\frac{\partial D}{\partial t} + J\right).\boldsymbol{ds} = 0 \longrightarrow 5$$

Equations 4 and 5 suggest that $\left(\frac{\partial D}{\partial t} + J\right)$ may be regarded as the total current density for time varying fields.

Since 'D' is the displacement density, $\frac{\partial D}{\partial t}$ is known as the "**displacement current density** ".

By taking total current density as $\left(\frac{\partial D}{\partial t} + J\right)$ then the ampere's law becomes as

$$\nabla \times H = \frac{\partial D}{\partial t} + J \longrightarrow 6$$

Taking the divergence to the aove equation the inconsistency has beed removed.

Integration of above equation over a surface and application of Stoke's theorem leads to the integral form

$$\oint_L \mathbf{H}.\boldsymbol{dl} = \int_s \left(\frac{\partial D}{\partial t} + J\right).\boldsymbol{ds} \longrightarrow 7$$

The above equation states that the magnetomotive force around a closed path is equal to the total current enclosed by the path.

Example:

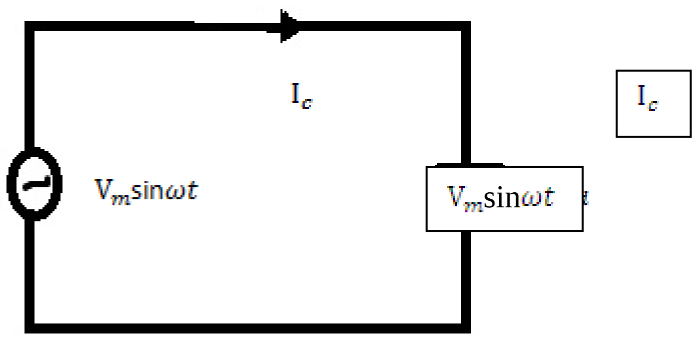
Show that the total displacement current between the condenser plates connected to same as the value of the charging current (conduction current).

Sol:-

The capacitance of a capacitor

$$C = \frac{\varepsilon A}{d} \longrightarrow 1$$

Where '**A**' is the plate area and ' **d** ' is the separation.



The conduction current is

$$i_c = \frac{\partial Q}{\partial t} = C\frac{dv}{dt}$$

$$i_c = \frac{CA}{d}.\frac{dv}{dt} \longrightarrow 2$$

The electric field in the dielectric is,

neglecting fringing

$$E = \frac{v}{d} \quad Hence\ D = \in E$$

$$D = \frac{\epsilon v}{d} \longrightarrow 3$$

The displacement current is

$$i_d = \int_s J.\boldsymbol{ds}$$

$$i_d = \int_s \frac{\partial D}{\partial t}.ds = \int_s \frac{\partial}{\partial t}\left(\frac{\epsilon v}{d}\right).ds$$

$$= \frac{\epsilon v}{d}\int_s \frac{\partial v}{\partial t}.ds = \frac{\epsilon A}{d}.\frac{\partial v}{\partial t}$$

$$i_d = i_c$$

Ratio of J_c to J_D

Some materials are neither good conductors nor perfect dielectrics, so that both conduction current and displacement current exist.

Assuming the time dependence $e^{j\omega t}$ for E,

The total current density is

$$J_t = J_C + J_D$$

$$= \sigma E + \frac{\partial D}{\partial t}$$

$$= \sigma E + \frac{\partial}{\partial t}(\in E)$$

$$= \sigma E + j\omega \in E$$

From which

$$\frac{J_C}{\text{[illegible]}} = \frac{\sigma E}{\text{[illegible]}}$$

The displacement current becomes increasingly important as the frequency increases.

Example:

In a material for which $\sigma = 5\ S/m$ and $\in_r = 1$, the electric field intensity $E = 250 \sin 10^{10} t\ V/m$. Calculate the conduction and displacement current density and the frequency at which they have equal magnitudes.

Sol:-

Given $\sigma = 5\ S/m$

$$\epsilon_r = 1$$

$$E = 250 sin 10^{10} t \ V/_m$$

$$J_c = \sigma E$$

$$= 5 \times 250 sin 10^{10} t \ A/_{m^2}$$

$$J_c = 1250 sin 10^{10} t \ A/_{m^2}$$

On the assumption that the field direction does not vary with time

$$J_D = \frac{\partial D}{\partial t} = \frac{\partial}{\partial t}(\epsilon_r \epsilon_r . 250 \sin 10^{10} t)$$

$$J_D = 22.1 \ cos 10^{10} t \ \ A/_{m^2}$$

$$For \ J_c = J_D$$

$$\sigma = \omega \in (or)$$

$$\omega = \frac{5}{8.854 \times 10^{-12}} = 5.65 \times 10^{11} \ \ rad/_{sec}$$

which is equivalent to a frequency

$$f = 8.854 \times 10^{10} \ Hz = 89.9 \ GHz$$

<u>Example:</u>

A copper wire carries a conduction current of 1 Amp. Determine the displacement current in the wire at 100 Hz. Take ϵ_o and conductivity = $5.8\times 10^{10}\ {}^{V}/_{m}$.

Sol:-

Given $i_C = 1\ Amp$ i_d =?

$f = 100\ MHz$

$= \epsilon_o$

$\sigma = 5.8\times 10^{7}\ /m$

We know that $\frac{J_C}{J_D} = \frac{\sigma E}{\omega \epsilon}$

for a copper wire of same length

$\frac{i_C}{[illegible]} = \frac{\sigma}{[illegible]}$

$\frac{1}{i_d} = \frac{5.8\times10^{7}}{2\pi\times100\times10^{6}\times\frac{1}{[illegible]}\times10^{-9}}$

$\frac{1}{[illegible]} = \frac{5.8\times 18}{10^{-8}}$

$i_d = 0.009578\times 10^{-8}\ Amp$

Word statement's of the Maxwell's equations for the time varying fields:

The magneto motive force around a closed path is equal to the conduction current plus the time derivative of the electric displacement through any surface bounded by the path

$$\oint_L \mathbf{H}.\boldsymbol{dl} = \int_s \left(J + \frac{\partial D}{\partial t} \right).\boldsymbol{ds}$$

The electro motive force around a closed path is equal to the time derivative of the magnetic displacement through any surface bounded by the path.

$$\oint_L \mathbf{E}.\boldsymbol{dl} = -\int_s \frac{\partial B}{\partial t}.\boldsymbol{ds}$$

The total electric displacement through the surface enclosing a volume is equal to the total charge within the volume.

$$\oint_s \mathbf{D}.\boldsymbol{ds} = \int_v \rho_v.dv$$

The net magnetic flux emerging through any closed surface is zero.

$$\oint_s \mathbf{B}.\boldsymbol{ds} = \mathbf{0}$$

Generalized forms of Maxwell's equations in differential form

$$\nabla \cdot \boldsymbol{D} = \rho_v \qquad Gauss's\ Law$$

$$\nabla \cdot \boldsymbol{B} = \mathbf{0} \qquad No\ existence\ of\ isolated\ magnetic\ charge$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \qquad Faraday's\ Law$$

$$\nabla \times H = \quad + \frac{\partial D}{\partial t} \qquad Ampere's\ circuital\ Law$$

Generalized forms of maxwell’s equations in integral form:

$$\oint_s \mathbf{D}.\boldsymbol{ds} = \int_v \rho_v . dv$$

$$\oint_s \mathbf{B}.\boldsymbol{ds} = \mathbf{0}$$

$$\oint_L \mathbf{E}.\boldsymbol{dl} = -\int_s \frac{\partial B}{\partial t}.\boldsymbol{ds}$$

$$\oint_L \mathbf{H}.\boldsymbol{dl} = \int_s \left(\mathrm{J} + \frac{\partial D}{\partial t} \right).\boldsymbol{ds}$$

Potential for time varying fields:

Maxwell’s equations in time varying differential form are

$$\nabla \times H = \mathbf{J} + \epsilon \dot{\mathrm{E}} \longrightarrow 1$$

$$\nabla \times E = -\mu \dot{\mathrm{H}} \longrightarrow 2$$

$$\nabla \cdot E = \frac{\rho}{\epsilon} \longrightarrow 3$$

$$\nabla \cdot H = 0 \longrightarrow 4$$

J and ρ are the source current density and the source charge density respectively and are related by the equation of continuity.

$$\nabla \cdot J = -\dot{\rho} \longrightarrow 5$$

Equation (4) is satisfied if **H** is represented as the curl of some vector. This leads to the following definition for the vector potential **A**

$$\mu H = \nabla \times A \longrightarrow 6$$

Substituting equation (6) into equation (2) gives

$$\nabla \times E = -\mu \dot{H}$$

$$= -\frac{\mu (\nabla \times \dot{A})}{\mu}$$

$$= -\nabla \times \dot{A}$$

$$\nabla \times (E + \dot{A}) = 0 \longrightarrow 7$$

equation (7) is satisfied if $E + \dot{A}$ is represented as the gradient of a scalar.

Setting $E + \dot{A}$ equal to ∇V

Thus the electric field strength may be expressed as

$$E = -\nabla V - \dot{A} \longrightarrow 8$$

The equation (6) and (8) shows how the field quantities **E** and **H** may be expressed in terms of a vector potential '**A**' and a scalar potential ' **V** and these expressions satisfies two of Maxwell's equation (2) and (4).

The remaining two of Maxwell's equations may be used to derive differential equations for the potential functions.

Substituting equation (6) & (8) into equation (1) gives

$$\frac{1}{\mu}[\nabla \times (\nabla \times A)] = \in [-\nabla V - \ddot{A}] +$$

$$\nabla \cdot (\nabla \cdot A) - \nabla^2 A = -\mu \in \nabla \dot{V} - \mu \in \ddot{A} + \mu J$$

$$\nabla^2 A - \mu \in \ddot{A} = \nabla \cdot (\nabla \cdot A) + \mu \in \nabla \dot{V} - \mu J$$

If the divergence of **A** is set equal to - $\mu \in \dot{V}$, the above equation becomes uncoupled and reduce to standard wave equations.

$$\nabla^2 A - \mu \in \ddot{A} = -\mu J \longrightarrow 9$$

By Substituting equation (8) into equation (3) gives

$$\nabla \cdot (-\nabla V - \dot{A}) = \frac{\rho}{\epsilon}$$

$$-\nabla^2 V - \nabla \cdot \dot{A} = \frac{\rho}{\epsilon}$$

$$\nabla^2 V + \nabla \cdot \dot{A} = -\frac{\rho}{\epsilon}$$

$$\nabla^2 V - \mu \in \ddot{V} = -\frac{\rho}{\epsilon} \longrightarrow 10$$

The particular choice used above that

$$\nabla \cdot A = \mu \in \dot{V}$$

is known as “Lorentz gauge condition”

Boundary conditions

If the field exists in a region consisting of two different media, the conditions that the field must satisfy at the interface separating the media are called "Boundary condition".

These boundary conditions are helpful in determining the field on one side of the boundary if the field on the other side is known.

We shall consider the boundary conditions at the interface separating

1. dielectric (ϵ_{r1}) and dielectric (ϵ_{r2})
2. conductor and dielectric
3. conductor and free-space

To determine the boundary conditions we need to use the Maxwell's equations.

$$\oint_{l} \mathbf{E}.\boldsymbol{dl} = 0$$

$$\oint_{s} \mathbf{D}.\boldsymbol{ds} = Q_{enclosed}$$

And also we need to decompose the electric field intensity **E** into two orthogonal components.

$$E = E_t + E_n$$

Di-Electric - Di-Electric boundary conditions:\

Consider the electric field existing in a region consisting of two different Dielectrics with permittivity

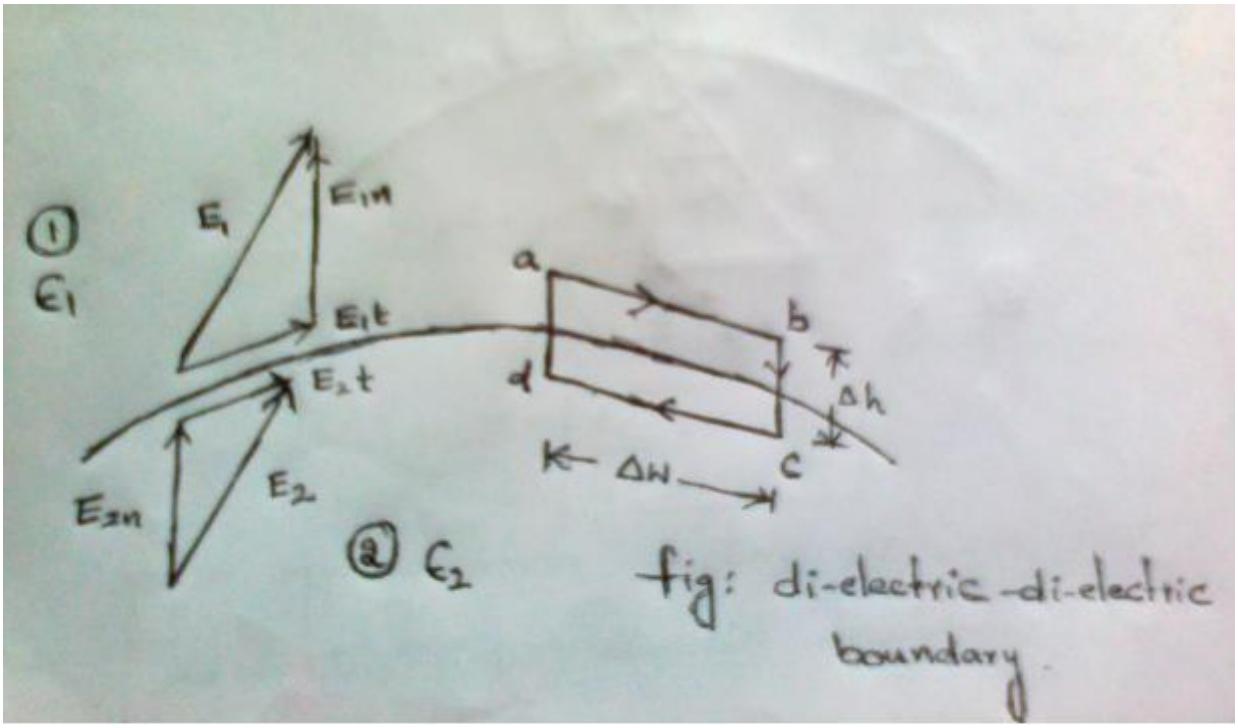
$$\varepsilon_1 = \varepsilon_o \varepsilon_{r1}, \quad \varepsilon_2 = \varepsilon_o \varepsilon_{r2}$$

Let 'E' is the electric field intensity and E_{tan} & E_n are the tangential and normal components of **'E'.**

E_1 and E_2 are the field intensity's in media ① and ② can be de composed as

$E_1 = E_{1t} + E_{1n}$

$E_2 = E_{2t} + E_{2n}$



If we consider a closed path abcda shown in fig. the amount of work done in carrying a unit

Charge completely around a closed path is zero. i.e. $\oint E.dl$=0

$$\oint_a^b E \cdot dl + \oint_b^c E \cdot dl + \oint_c^d E \cdot dl + \oint_d^a E \cdot dl = 0$$

Assuming that the path is very small with respect to the variation of E, we obtain

$$\boldsymbol{E_{1t}}\nabla \mathbf{w} + \left(-\boldsymbol{E_{1n}} \Big/ \frac{\nabla \mathrm{h}}{2} - \boldsymbol{E_{2n}} \Big/ \frac{\nabla \mathrm{h}}{2} \right) - \boldsymbol{E_{2t}}\nabla \mathbf{w} + \left(\boldsymbol{E_{2n}} \Big/ \frac{\nabla \mathrm{h}}{2} + \boldsymbol{E_{1n}} \Big/ \frac{\nabla \mathrm{h}}{2} \right) = 0$$

$$\boldsymbol{E_{1t}} = \boldsymbol{E_{2t}}$$

→ Thus the tangential components of E are the same on the two sides of boundary.

In other words $\boldsymbol{E_t}$ undergoes no change on the boundary and it is said to be continuous across the boundary.

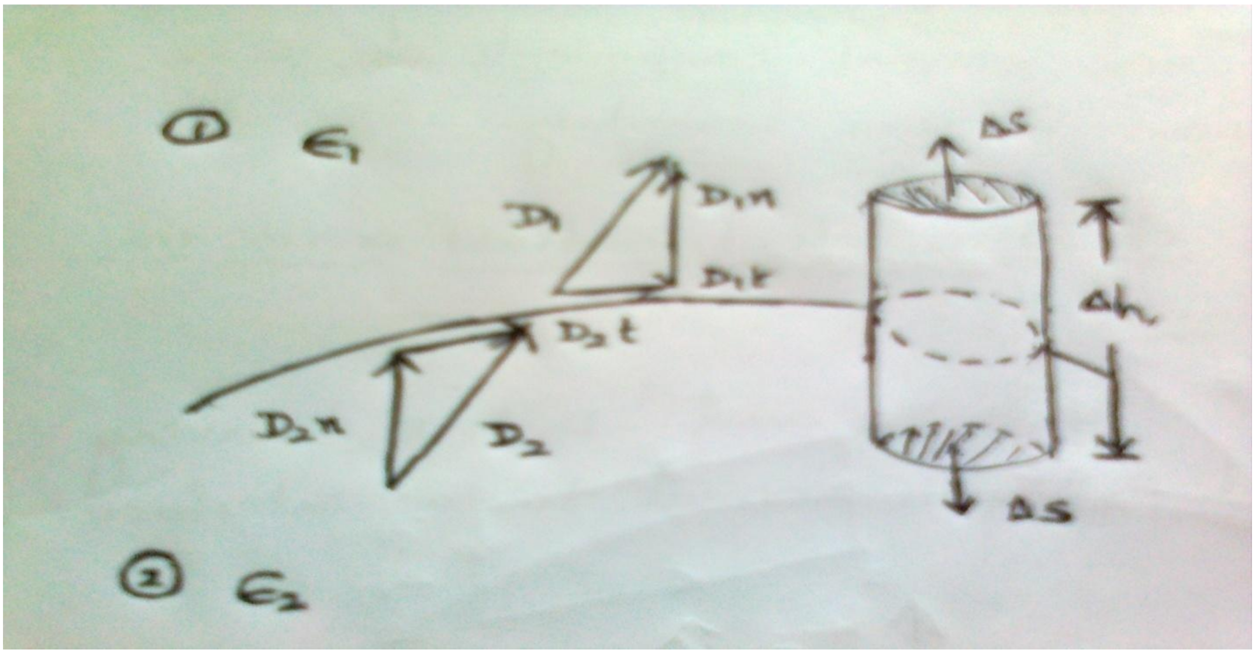
→ Since $\mathbf{D} = \varepsilon\mathbf{E} = \boldsymbol{D_t} + \boldsymbol{E_n}$

$$\boldsymbol{E_{1t}} = \boldsymbol{E_{2t}}$$

$$\frac{D_{1t}}{-} = \frac{D_{2t}}{-}$$

Thus the D_t undergoes some change across the interface. Hence D_t is said to be continuous across the boundary.

→Consider a small pill box (or) Gaussian surface shown in fig below.



→Let Δs is the surface area of the top/bottom.

From Gauss law $\therefore$ $\oint_s \mathbf{D} \cdot ds = Q_{enclosed}$

$$\Delta Q = D_{1n} \nabla S - D_{2n} \Delta S$$

→We know that $\Delta Q = \rho_s \Delta S$

Where ρ_s → Surface charge density

$$\rho_s \Delta S = (D_{1n} - D_{2n})\Delta S$$

$$\boxed{D_{1n} - D_{2n} = \rho_s \Delta s}$$

If no free charges exist at the interface i.e. ρ_s **=0**

$\therefore$ the normal component of D is continuous across the interface.

$$D_{1n} = D_{2n}$$

→ Since $\mathbf{D} = \varepsilon\mathbf{E}$

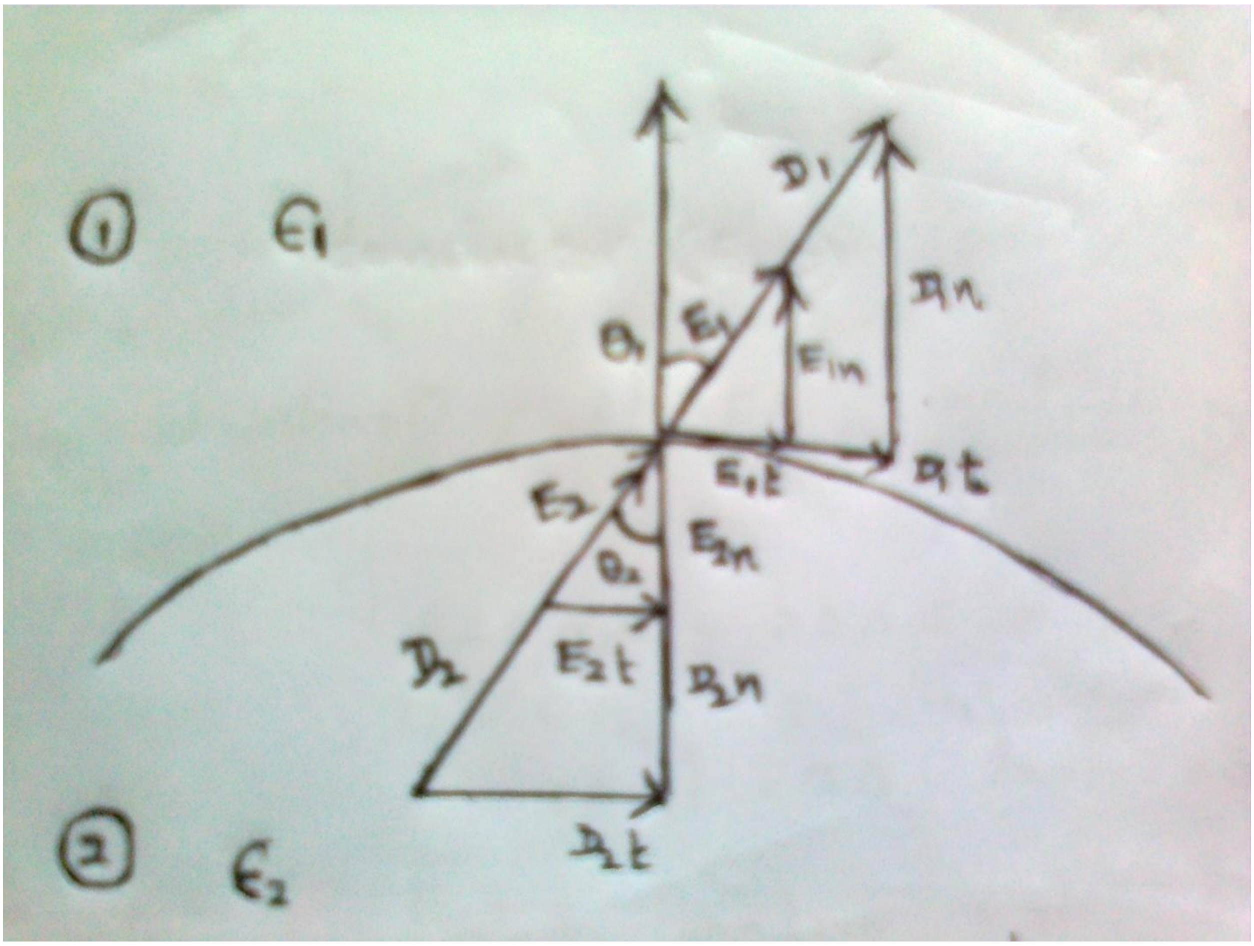
$$\therefore \quad D_{1n} = D_{2n}$$

$$\varepsilon_1 E_{1n} = \varepsilon_2 E_{2n}$$

Shows the normal components of E are discontinuous at the boundary.

Refraction of the electric field across the interface

→ Consider $\boldsymbol{D_1}$ or $\boldsymbol{E_1}$ **and** $\boldsymbol{D_2}$ *or* $\boldsymbol{E_2}$ making angles $\boldsymbol{\theta_1}$ *and* $\boldsymbol{\theta_2}$ normal to the interface.



→ The tangential components of electric field are continuous at the interface.

$$\therefore \quad E_{1t} = E_{2t}$$

From the above fig

$$E_{1t} = E_1 \sin\theta_1$$

$$E_{2t} = E_2 \sin\theta_2$$

$$\therefore \quad E_{1t} = E_{2t}$$

$$E_1 \sin\theta_1 = E_2 \sin\theta_2 \longrightarrow \quad 1$$

→ The normal components of D are continuous across the interface.

i.e. $D_{1n} = D_{2n}$

From the fig $D_{1n} = D_1 \cos\theta_1$

$D_{2n} = D_2 \cos\theta_2$

$\therefore \quad D_{1n} = D_{2n}$

$$D_1 \cos\theta_1 = D_2 \cos\theta_2$$

$$\varepsilon_1 E_1 \cos\theta_1 = \varepsilon_2 E_2 \cos\theta_2 \longrightarrow \quad 2$$

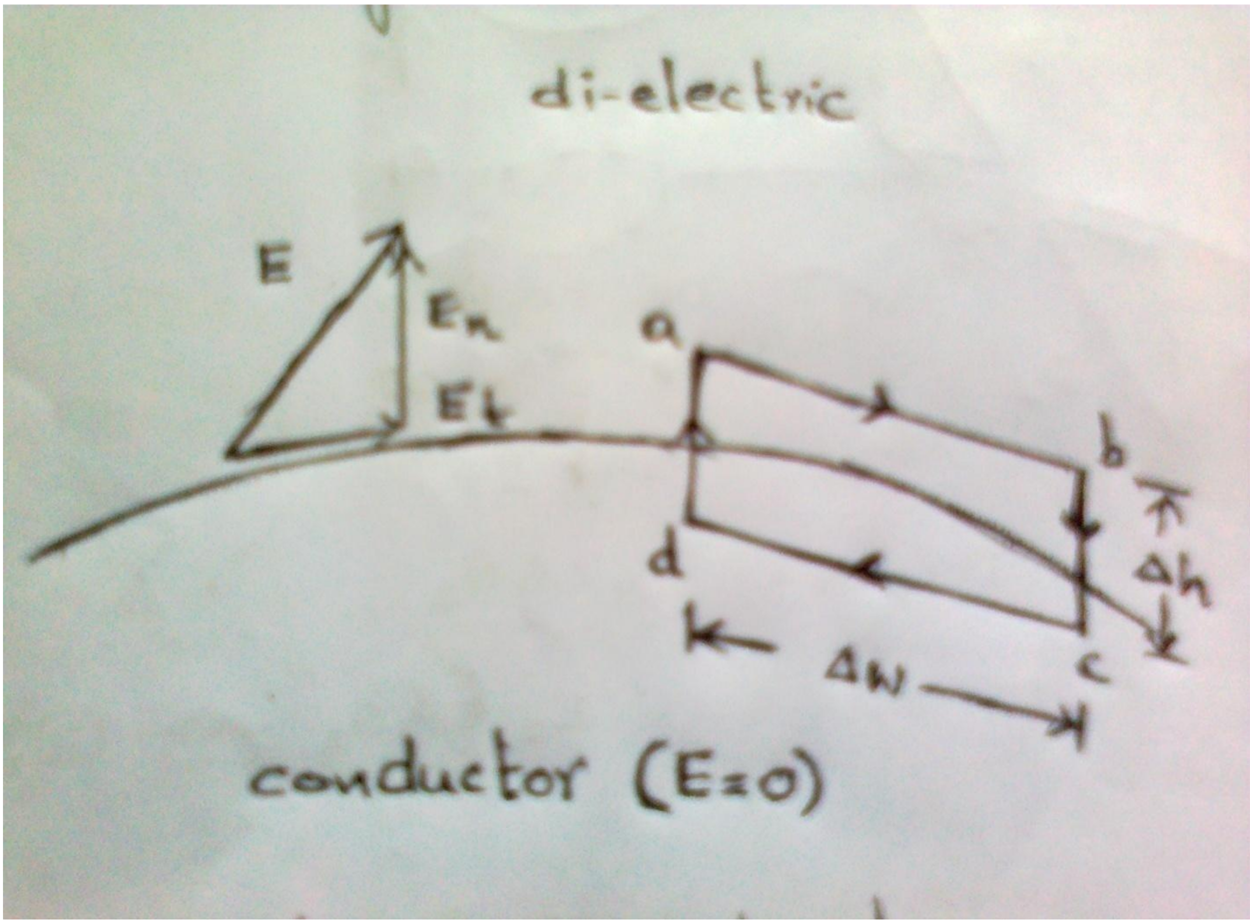
$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_1}{\varepsilon_2}}$$

$$\frac{\tan\theta_1}{\varepsilon_1} = \frac{\tan\theta_2}{\varepsilon_2}$$

This is the "Law of refraction "of the electric field at a boundary free of charge.

Conductor – dielectric boundary conditions

→ To determine the boundary conditions for a Conductor – dielectric interface we incorporate the fact that the electric field **E = 0** inside the conductor



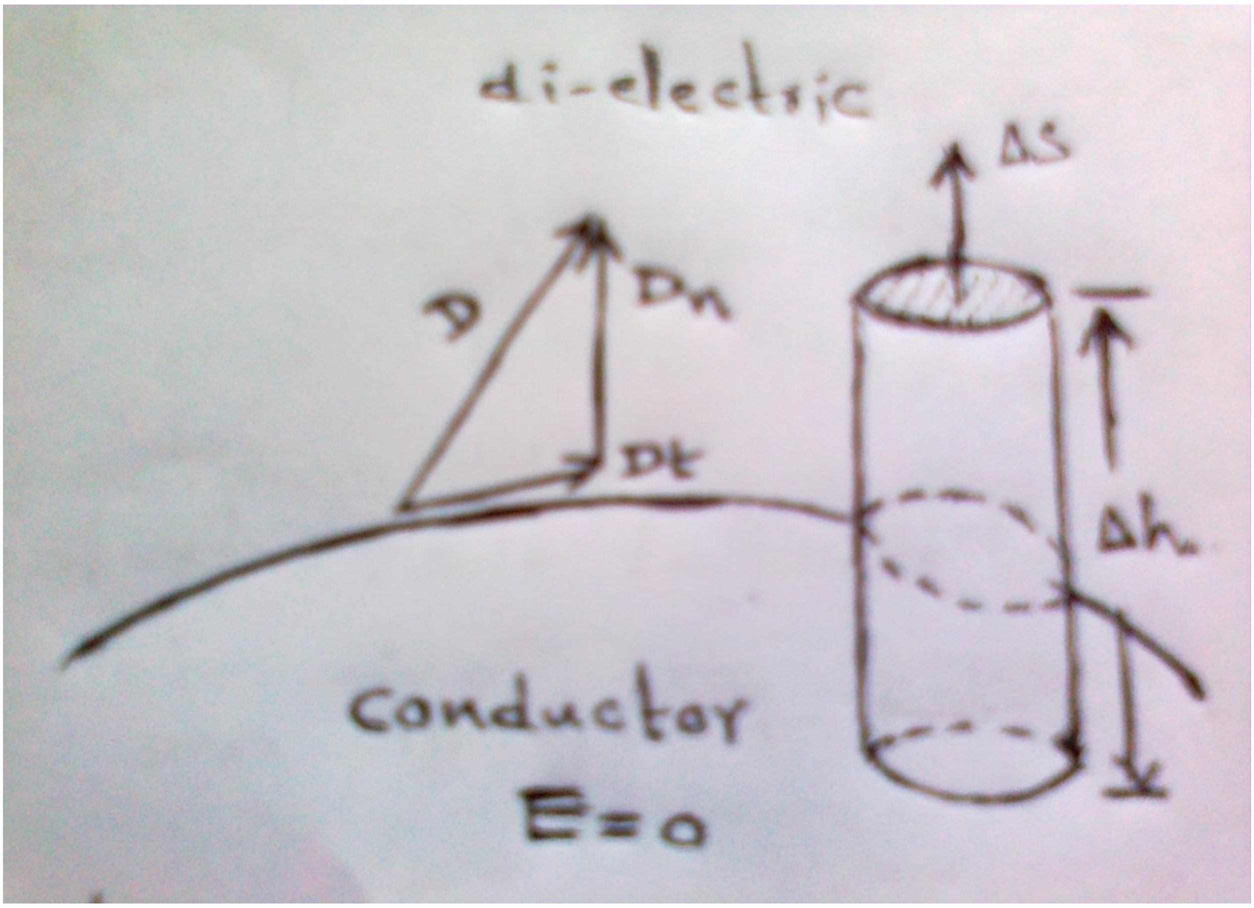
→ By considering a closed path abcda as shown in fig. below

$$\oint_l \boldsymbol{E.dl} = \mathbf{0}$$

$$\oint_a^b E.dl + \oint_b^c E.dl + \oint_c^d E.dl + \oint_d^a E.dl = 0$$

$$\boldsymbol{E_t \nabla \mathrm{w}} = \mathbf{0}$$

$\boxed{E_t = 0}$ $\boxed{D_t = 0}$



→ Similarly by applying Gauss's law to the pill box shown in above fig. we get

$$\oint \mathbf{D}.ds = Q_{enclosed}$$

$$D_n \, \Delta S - 0 = \Delta Q$$

$$D_n \, \Delta S = \Delta Q$$

$$D_n \, \Delta S = \rho_s \, \Delta S$$

$$\boxed{D_n = \rho_s}$$

$$\varepsilon_o \varepsilon_r E_n = \rho_s$$

$$E_n = \frac{\rho_s}{}$$

Conductor – free space boundary conditions

→ This is a special case of the conductor dielectric conditions.

The boundary conditions at the interface between a conductor and free space are

$$D_t = \varepsilon_o E_t = 0$$

$$D_n = \varepsilon_o E_t = \rho_s$$

Magnetic boundary conditions

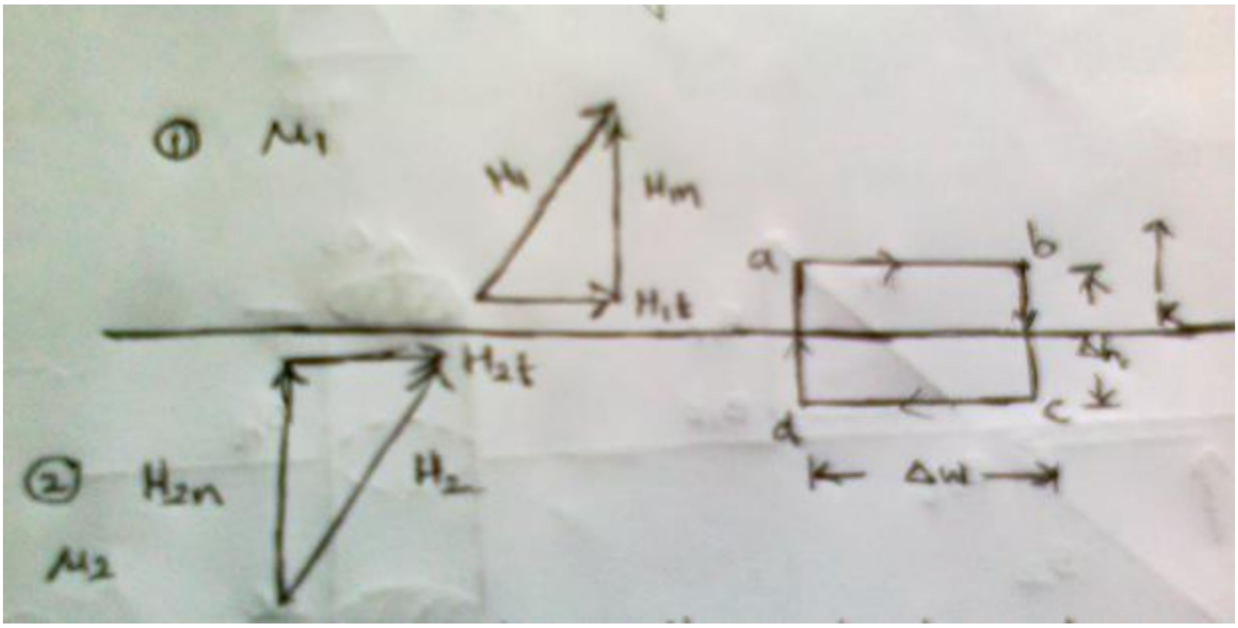
→ Magnetic boundary conditions are the conditions that H or B field satisfy at the boundary between two different media.

→ To determine the boundary conditions we need to use Maxwell's equations.

$$\oint \mathbf{B}.d\boldsymbol{s} = 0$$

$$\oint \mathbf{H}.d\boldsymbol{l} = I_{enclosed}$$

→ Consider a boundary between two isotropic homogeneous linear materials with permiabilities with linear perm abilities μ_1 and μ_2 as shown in fig. below



→ Consider a closed path abcda shown in fig.

$$\oint_a^b H.dl + \oint_b^c H.dl + \oint_c^d H.dl + \oint_d^a H.dl = \boldsymbol{K}.\nabla\mathbf{w}$$

$$\boldsymbol{H}_{1t}\nabla\mathbf{w} + \left(-\boldsymbol{H}_{1n}/_{\nabla h} - \boldsymbol{H}_{2n}/_{\nabla h} \right) + \boldsymbol{H}_{1t}\nabla\mathbf{w} + \boldsymbol{H}_{2n}/_{\nabla h} + \boldsymbol{H}_{1n}/_{\nabla h} = \boldsymbol{K}.\nabla\mathbf{w}$$

→ $$\boldsymbol{H}_{1t} - \boldsymbol{H}_{2t} = \boldsymbol{K}$$

$$\boxed{\boldsymbol{H}_{1t} = \boldsymbol{H}_{2t}}$$

Where K is the sheet current density in $A/_m$.

$$If\ K = 0$$

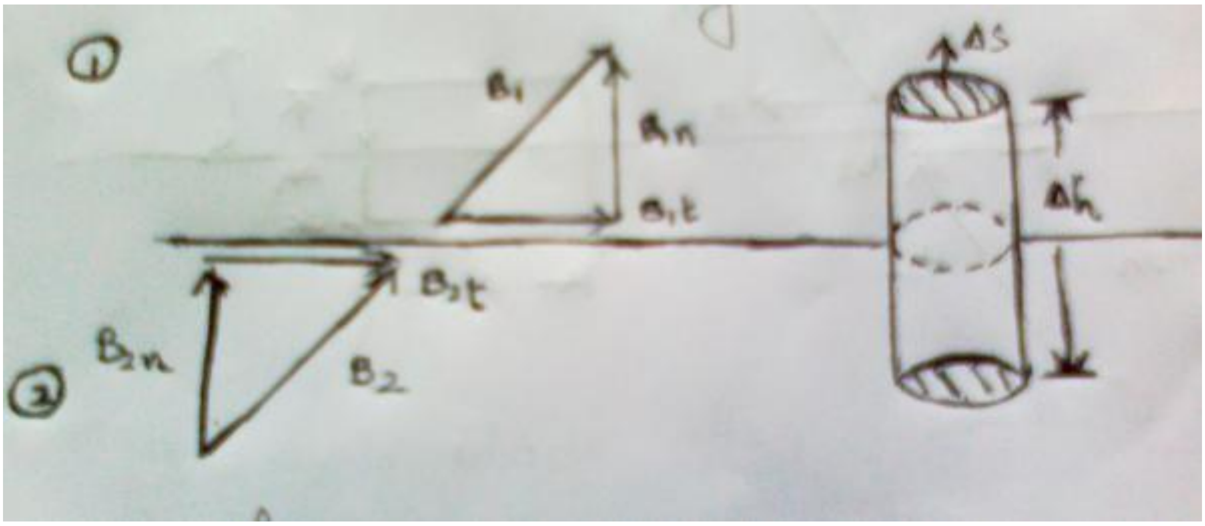
Thus the tangential components of H are continuous.

$$\frac{B_{1t}}{\ } = \frac{B_{2t}}{\ }$$

$$\frac{B_{1t}}{\ } = \frac{\mu_1}{\ }$$

& tangential components of B are discontinuous at the boundary.

→ Consider a small pill box (or) Gaussian surface shown in fig.



$$\oint_s \mathbf{B}.ds = 0$$

$$B_{1n}\,\Delta S - B_{2n}\,\Delta S = 0$$

$$B_{1n} = B_{2n}$$

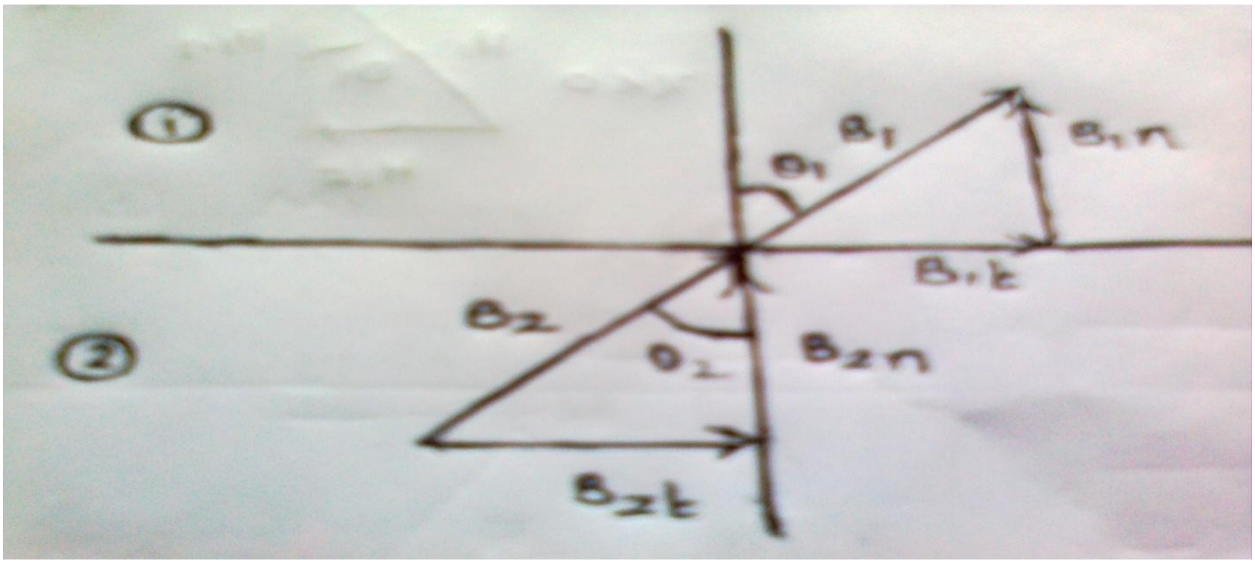
$$\mu_1 H_{1n} = \mu_2 H_{2n}$$

$$\boxed{\frac{H_{1n}}{H_{2n}} = \frac{\mu_2}{\mu_1}}$$

The normal components of B are continuous at the boundary and the normal components of H are discontinuous.

Law of refraction for magnetic flux lines at the interface:

→ Consider B_1 or H_1 and B_2 or H_2 making angles θ_1 *and* θ_2 normal to the interface.



→ The magnetic boundary conditions are

$$B_{1n} = B_{2n}$$

$$H_{1t} = H_{2t}$$

→ From the above fig $B_1 \cos\theta_1 = B_{1n}$

$$B_2 \cos\theta_2 = B_{2n}$$

$$H_1 \sin\theta_1 = H_{1t}$$

$$H_2 \sin\theta_2 = H_{2t}$$

Law of refraction

For magnetic flux lines

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2}$$

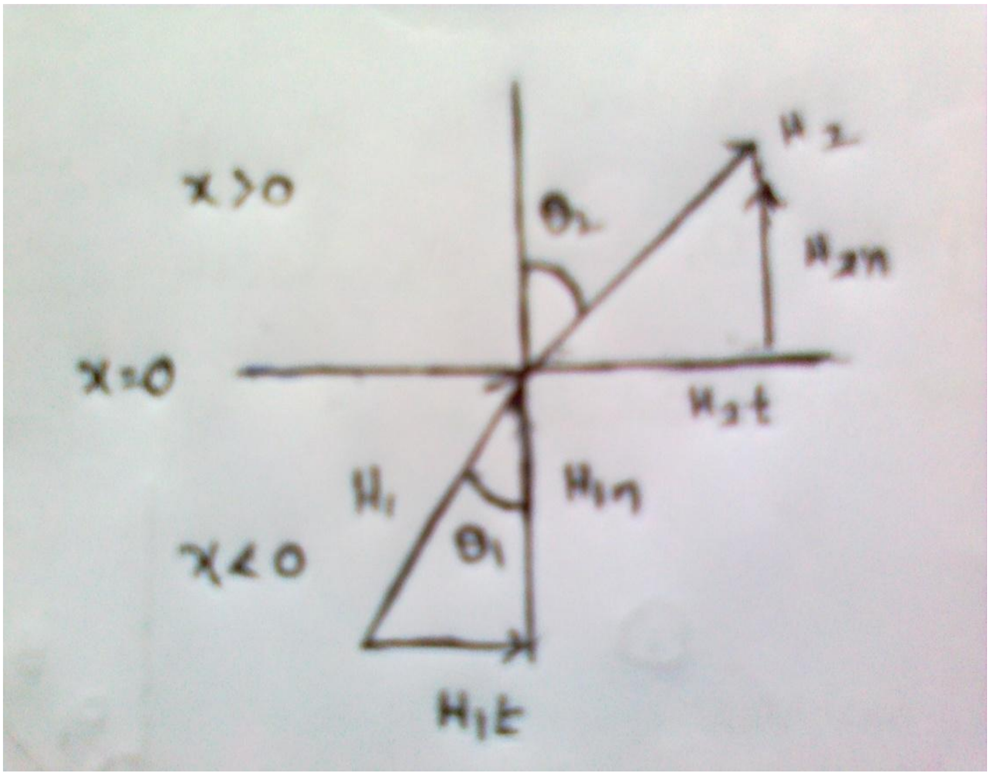
→ $$H_{2n} = \frac{3}{5} H_{1n}$$

$$H_{2n} = \frac{3}{5} * 4i_x$$

$$H_{2n} = \frac{12}{5} * i_x = 2.4i_x$$

$$H_2 = H_{2t} + H_{2n}$$

$$H_2 = 2.4i_x + 3i_y + 6i_z \; A/m.$$



From the fig $\cos\boldsymbol{\theta}_1 = \frac{\boldsymbol{H}_{1n}}{\boldsymbol{H}_1}$

$$\cos\boldsymbol{\theta}_1 = \frac{\mathbf{4}}{\sqrt{\mathbf{61}}} = \frac{\mathbf{4}}{\mathbf{7.180}}$$

$$\boldsymbol{\theta}_1 = \mathbf{cos}^{-1}\,\mathbf{0.5121}$$

$$\boldsymbol{\theta}_1 = \mathbf{59.19}^{\circ}$$

$$\cos\boldsymbol{\theta}_2 = \frac{\boldsymbol{H}_{2n}}{\boldsymbol{H}_2}$$

$$\cos\boldsymbol{\theta}_2 = \frac{\mathbf{204}}{\text{[illegible]}}$$

$$\boldsymbol{\theta}_2 = \mathbf{70.3}^{\circ}$$

→ θ_1 *and* θ_2 *are the angles of* H_1 *and* H_2 with normal to the interface.

→ Angles that H_1 *and* H_2 make with the interface are 90-$\boldsymbol{\theta}_1$ **&** 90-$\boldsymbol{\theta}_2$ **i.e.** $\mathbf{30.81}^{\circ}$ **and** $\mathbf{19.7}^{\circ}$

<u>Problem 1</u>: In a medium of $\mu_r = 2$ find E and B and displacement current density if $H = 25\sin(2\times10^8 t + 6x)\,\boldsymbol{i}_y\ {}^{mA}/_{m}$.

<u>SOL</u>: Given $\mu_r = 2$

$$H = 25\sin(2\times10^8 t + 6x)\,\boldsymbol{i}_y\ {}^{mA}/_{m}$$

$\mathrm{B} = \mu H$

$$B = \mu_o \mu_r H$$

$$B = 50\mu_o \sin(2\times10^8 t + 6x)\, \boldsymbol{i}_y \ {}^{mwb}/_{m^2}$$

$$\nabla \times E = \frac{-\partial B}{\partial t}$$

$$\nabla \times E = 50\mu_o \cos(2\times10^8 t + 6x) \times 2\times10^8 \boldsymbol{i}_y$$

$$\nabla \times E = \mu_o \cos(2\times10^8 t + 6x) \times 10^{10} \boldsymbol{i}_y$$

$$\begin{vmatrix} \boldsymbol{i}_x & \boldsymbol{i}_y & \boldsymbol{i}_z \\ \frac{-\partial B}{\partial x} & \frac{-\partial B}{\partial y} & \frac{-\partial B}{\partial z} \\ \boldsymbol{E}_x & \boldsymbol{E}_y & \boldsymbol{E}_z \end{vmatrix} = \mu_o \cos(2\times10^8 t + 6x) \times 10^{10} \boldsymbol{i}_y$$

$$-\boldsymbol{i}_y \left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right) = \mu_o \cos(2\times10^8 t + 6x) \times 10^{10} \boldsymbol{i}_y$$

$$\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} = \mu_o \cos(2\times10^8 t + 6x) \times 10^{10}$$

$$\boldsymbol{E}_z = \frac{\mu_o}{6} \sin(2\times10^8 t + 6x)\, {}^{mv}/_{m}$$

→ $$\nabla \times H = \frac{\partial D}{\partial t} + J$$

$If\ J = 0 \qquad \nabla \times H = \frac{\partial D}{\partial t} = displacement\ current\ density$

$$\frac{\partial D}{\partial t} = \begin{vmatrix} \boldsymbol{i}_x & \boldsymbol{i}_y & \boldsymbol{i}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \boldsymbol{0} & \boldsymbol{H}_y & \boldsymbol{0} \end{vmatrix}$$

$$\frac{\partial D}{\partial t} = \frac{\partial}{\partial x}(25 \sin(2\times10^8 t + 6x)$$

$\frac{\partial D}{\partial t} = 25 \times 6 \cos(2 \times 10^8 t + 6x)$

$\frac{\partial D}{\partial t} = 150 \cos(2 \times 10^8 t + 6x)\ {}^{A}/_{m^2}$

EM Wave Characteristics

→ The electric and magnetic fields can move through space as waves.

→ In general waves are means of transporting energy or information.

→ A wave is a function of both space and time.

Maxwell's equation for free space:

→ For free space (perfect dielectric medium) containing no charges and conduction currents

$i.e. J = 0, \rho_v = 0$

$\epsilon = \epsilon_o \ And\ \mu = \mu_o$

→ The Maxwell equations are:

Differential form:-

$$\nabla \cdot D = \rho_v = 0 \longrightarrow \quad 1$$

$$\nabla \times E = -\frac{\partial B}{\partial t} = -\mu_o \frac{\partial H}{\partial t} \longrightarrow \quad 2$$

$$\nabla \times H = \frac{\partial D}{\partial t} + J$$

For free space J = 0

$$\nabla \times H = \frac{\partial D}{\partial t} = \epsilon_o \frac{\partial E}{\partial t} \longrightarrow 3$$

$$\nabla \cdot B = 0 \longrightarrow 4$$

Integral form:-

$$\oint_s D \cdot ds = 0$$

$$\oint_s B \cdot ds = 0$$

$$\oint_L E \cdot dl = -\int_s \frac{\partial B}{\partial t} \cdot ds$$

$$\oint_L H \cdot dl = \int_s \frac{\partial D}{\partial t} \cdot ds$$

Wave equations for electric field (E) in conducting medium:

→Consider the Maxwell's equation

$$\nabla \times E = -\frac{\partial B}{\partial t} \longrightarrow 1$$

→ Taking curl on both sides

$$\nabla \times (\nabla \times E) = \nabla \times \left(-\frac{\partial B}{\partial t}\right)$$

$$\nabla(\nabla\cdot \mathrm{E}) - \nabla^2 \mathrm{E} = -\nabla \times \left(+\mu \frac{\partial H}{\partial t}\right)$$

$$= -\mu\nabla \times \left(\frac{\partial H}{\partial t}\right)$$

$$\nabla(\nabla\cdot \mathrm{E}) - \nabla^2 \mathrm{E} = -\mu \frac{\partial}{\partial t}(\nabla \times H) \longrightarrow 2$$

But $\nabla \times H = \frac{\partial D}{\partial t} + \mathrm{J}$

Substituting this in above equation we get

$$\nabla(\nabla\cdot \mathrm{E}) - \nabla^2 \mathrm{E} = -\mu \frac{\partial}{\partial t}\left(\frac{\partial D}{\partial t} + \mathrm{J}\right)$$

We have $\nabla.\mathrm{D} = \rho_v$

$\nabla.\in E = \rho_v$

$\nabla.E = \frac{\rho_v}{\in}$

Inside to the conductor the charge density is Zero i.e. $\rho_v = 0.$

∴ No net charge within a conductor.

∴ $\nabla\cdot \mathrm{E} = 0$

Substituting this in above equation we get

$$-\nabla^2 \mathrm{E} = -\mu \frac{\partial}{\partial t}\left(\frac{\partial D}{\partial t} + \mathrm{J}\right)$$

Putting $J = \sigma E$ and $D = \in E$

The above equation becomes as

$$\nabla^2 E = \mu \frac{\partial}{\partial t}(\sigma E + \epsilon \frac{\partial E}{\partial t})$$

$$\nabla^2 E = \mu \sigma \frac{\partial E}{\partial t} + \mu \epsilon \frac{\partial^2 E}{\partial t^2}$$

This is called as wave equation for conducting medium for electric field.

Wave equations for electric field (E) in conducting medium for magnetic fields (H):

→Consider the Maxwell's equation

$$\nabla \times H = \frac{\partial D}{\partial t} + J \longrightarrow 1$$

→ Taking curl on both sides

$$\nabla \times (\nabla \times H) = \nabla \times (J + \frac{\partial D}{\partial t})$$

$$\nabla(\nabla \cdot H) - \nabla^2 H = \nabla \times (\sigma E + \epsilon \frac{\partial E}{\partial t})$$

→ We have the Maxwell's equation

$$\nabla . B = 0 \Rightarrow \nabla . H = 0$$

$$-\nabla^2 H = \sigma(\nabla \times E) + \nabla \times \epsilon \frac{\partial E}{\partial t}$$

$$-\nabla^2 H = \sigma(\nabla \times E) + \epsilon \frac{\partial}{\partial t}(\nabla \times E)$$

→ We have the Maxwell's equation

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$-\nabla^2 \mathrm{H} = \sigma(-\frac{\partial B}{\partial t}) + \in \frac{\partial}{\partial t}(-\frac{\partial B}{\partial t})$$

$$-\nabla^2 \mathrm{H} = -\sigma\frac{\partial B}{\partial t} - \in \frac{\partial^2 B}{\partial t^2}$$

$$\nabla^2 \mathrm{H} = \mu\sigma(\frac{\partial H}{\partial t}) + \mu \in \frac{\partial^2 H}{\partial t^2}$$

This is called as wave equation for conducting medium for magnetic field.

Wave equations for perfect dielectric medium or free space

For a perfect dielectric medium or non-conducting medium like free space containing no charges and no conduction currents

i.e. $\sigma = 0$ and $\rho_v = 0$

$J = \sigma \mathrm{E} \quad \mu = \mu_0 \quad \epsilon = \epsilon_0$

The Wave equations for free space are:

$$\nabla^2 \mathrm{E} = \mu_0 \in_o \frac{\partial^2 E}{\partial t^2} \textit{ for static electric fields}$$

$$\nabla^2 \mathrm{H} = \mu_0 \in_o \frac{\partial^2 H}{\partial t^2} \textit{ for static magnetic fields}$$

From the above equations

$$\nabla^2 \mathrm{D} = \mu_0 \in_o \frac{\partial^2 D}{\partial t^2}$$

$$\nabla^2 B = \mu_0 \in_o \frac{\partial^2 B}{\partial t^2}$$

Expression for velocity of light

we have

$$\mu_0 = 4\pi * 10^{-7} \ {}^{H}/{}_{m}$$

$$\in_o = \frac{1}{36\pi} * 10^{-9} \ {}^{F}/{}_{m}$$

$$\mu_0 \in_o = \frac{10^{-16}}{9} = \frac{1}{9 * 10^{16}}$$

We know that the velocity of light in free space is $3 * 10^8 \ {}^{m}/{}_{sec}$

$$c = 3 * 10^8 \ {}^{m}/{}_{sec}$$

$$c^2 = 3 * 10^8 \ {}^{m}/{}_{sec}$$

Substituting in the above equation

$$\mu_0 \in_o = \frac{1}{C^2}$$

$$c = \frac{1}{\sqrt{\mu_0 \in_o}}$$

Therefore the wave equations for free space are

$$\nabla^2 \mathrm{E} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 \mathrm{H} = \frac{1}{c^2}\frac{\partial^2 H}{\partial t^2}$$

these differential equations represent waves which propagate through free space with the velocity of light.

Wave: A wave is a function of both space and time

Plane Wave: A plane wave is defined as a wave for which the phase is the same at all points in a plane perpendicular to the direction of propagation of the wave.

Wave equation is in the form of

$\nabla^2 \mathrm{E} = \frac{1}{v^2}\frac{\partial^2 E}{\partial t^2}$ v is the wave velocity

Is a partial differential equation of the second order.

In one dimension, i.e by assuming E & H are considered to be independent of two dimensions say y & z, then the above equations becomes as

$$\frac{\partial^2 E}{\partial t^2} = \frac{1}{v^2}\frac{\partial^2 E}{\partial t^2}$$

The solution of the above equation is of the form

$E = f1(x - vt) + f2(x + vt)$

Where and are any functions such as $x \pm vt, sink(x \pm vt), cosk(x \pm vt)$ and $e^{(x \pm vt)}$ etc. where k is a constant.

The functions $f1(x - vt)\ and\ f2(x + vt)$ describe a wave mathematically, the variation of the wave confined to one dimension in space.

<u>Wave: -</u> A wave may be defined as following way:

If a physical phenomenon that occurs at a place at a given time is reproduced at other places at later times, the time delay being proportional to the space separation from the first location, then the groups of phenomena constitute a "wave".

The general solution of a wave equation consists of two waves, one travelling to the right (away from the source), and the other travelling to the left(back towards the source).

Sinusoidal time variations

Periodic variations can always be analyzed in terms of sinusoidal variations with fundamental and harmonic frequencies.

Electric and Magnetic fields which vary sinusoidally with time

$$E = E_0 sinwt$$

$$E = E_0 coswt$$

We shall assume that the fields are time harmonic “A time-harmonic fields is one that varies periodically or with time”.

Sinusoidal are easily expressed in phasors, which are more convenient to work with.

Maxwell’s equations for fields in phasor form

Let **E** be the time varying quantity which us either a function of sine or cosine.

The phasor of **E** is obtained by dropping R_e and suppressing e^{jwt}

$E = E_0 \cos\,(wt + \varphi) or$

$E = E_0 \sin(wt + \varphi).$

$$w = 2 * \pi * f \; ^{angular\, frequnency}$$

φ =$phase\ angle$

For complex quantity

$$e^{j(wt+\varphi)} = \cos(wt + \varphi) + j\,\sin(wt + \varphi)\cdot$$

taking the real part of the above equation

$$E = E_0 \mathrm{Re}\left(e^{j(wt+\varphi)}\right)$$

Dropping R_e & suppressing e^{jwt} gives

$$E = E_0 e^{j(\varphi)} * e^{j(wt)}$$

$$E = E_0 e^{j(wt+\varphi)}$$

by taking partial derivative on both sides with respect to time

$$\frac{\partial \mathrm{E}}{\partial t} = jw * E_0 e^{j(wt+\varphi)}$$

$$\frac{\partial \mathrm{E}}{\partial t} = jw * E$$

for field vectors varying harmonically with time, we may write

$$D = De^{j(wt+\varphi)}$$

$$\frac{\partial D}{\partial t} = jw * D$$

$$B = B_0 e^{j(wt+\varphi)}$$

$$\frac{\partial B}{\partial t} = jw * B$$

<u>Maxwell's equations for fields varying harmonically with time:</u>

Differential form:-

$$\nabla . \mathrm{D} = \dot{\rho}_v$$

$$\nabla . \mathrm{B} = 0$$

$$\nabla \times E = -\frac{\partial B}{\partial t} = \mathrm{jwB}$$

$$\nabla \times H = \frac{\partial D}{\partial t} + \mathrm{J}$$

$$\nabla \times H = \sigma \mathrm{E} + \mathrm{j}\omega \in E$$

$$\nabla \times H = (\sigma + \mathrm{j}\omega \in)E$$

Integral form:-

$$\oint_s \mathbf{D}.\boldsymbol{dl} = \int_s \rho_v . \boldsymbol{dv}$$

$$\oint_s \mathbf{B}.\boldsymbol{dl} = 0$$

$$\oint_L \mathbf{E}.\boldsymbol{dl} = -jw \int_s B.\boldsymbol{ds}$$

$$\oint_l \mathbf{H}.\boldsymbol{dl} = (\sigma + \mathrm{j}\omega \in) \int_s E.\boldsymbol{ds}$$

Wave equations for electric and magnetic fields:-

$$\nabla^2 \mathrm{E} = \mu\sigma\frac{\partial E}{\partial t} + \mu\epsilon\frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 \mathrm{E} = \mu\sigma jwE + \mu\epsilon(jw)(jw)E$$

$$\nabla^2 \mathrm{E} = \mathrm{jw}\mu(\sigma + \mathrm{j}\omega \in)E$$

Similarly

$$\nabla^2 \mathrm{H} = \mu\sigma\frac{\partial H}{\partial t} + \mu\epsilon\frac{\partial^2 H}{\partial t^2}$$

$$\nabla^2 \mathrm{H} = \mu\sigma jwH + \mu\epsilon(jw)(jw)H$$

$$\nabla^2 \mathrm{H} = \mathrm{jw}\mu(\sigma + \mathrm{j}\omega \in)H$$

For free space $\sigma = 0, \mu = \mu_0$ and $\in = \in_o$

$$\nabla^2 \mathrm{E} = -\mathrm{w}^2\mu_0 \in_o E$$

$$\nabla^2 \mathrm{H} = -\mathrm{w}^2\mu_0 \in_o H$$

Wave propagation

Let us consider a wave is represented by the equation

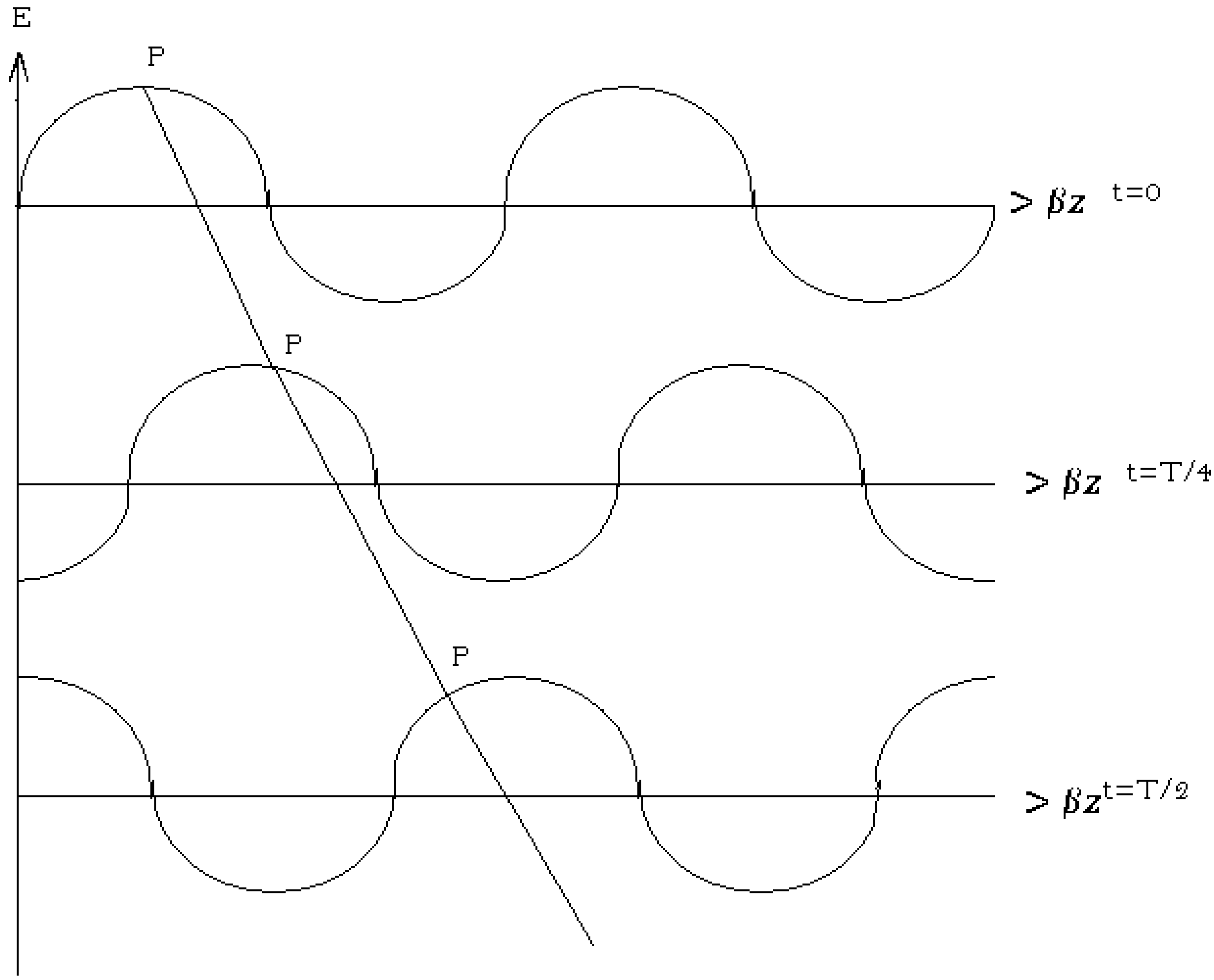
$$E = E_0 \sin(wt - \varphi)$$

where $(wt - \varphi)$is the phase of the wave it depends on time't' and space variable 'z' w=angular frequency β=phase constant

The above wave equation shows that a wave is travelling with a velocity v in the +z direction.

Proof:

We consider a fixed point P on the wave. We sketch the equation at times t=0, T/4 and T/2 as shown in the figure below



From the above fig. it is evident that as the wave advances with time , point P moves along +z direction.

Point P is a point of constant phase'

$$wt - \beta z = constant$$

$$\beta z = wt$$

$$\frac{\partial H}{\partial t} = \frac{w}{\beta} = v$$

The above equation shows that the wave travels with velocity v in the +z direction.

SUMMARY

1. A wave is a function of both time and space.
2. A negative sign in $(wt \pm \beta z)$ is associated with a wave propagating in the +z direction [forward travelling or positive –going wave]
3. A positive sign in $(wt \pm \beta z)$ indicates that a wave is travelling in the –z direction. [backward travelling or negative going wave.]

UNIFORM PLANE WAVE

If the magnitude and direction are the same in an infinite number of planes, all perpendicular to the direction of propagation of the wave, then the wave is termed as a "uniform plane wave".

For uniform plane waves:

Both the electric and magnetic fields are perpendicular to the direction of propagation.

The uniform plane wave is a transverse electromagnetic wave.

UNIFORM PLANE WAVE IS A TRANSVERSE ELECTRO MAGNETIC WAVE:-

Let us consider a wave which is propagating in the +ve direction of the x-axis.

For any wave travelling in the x-direction, (the y and z components of the field are zero) since the field is a function of x and t only.

Consider the wave equation for electric field is given by

$$\nabla^2 \mathrm{E} = \mu\epsilon \frac{\partial^2 E}{\partial t^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(1)$$

Let Ex, Ey and Ez be the components of E in the x, y, z direction.

$$\frac{\partial^2 E_x}{\partial x^2} = \mu\epsilon \frac{\partial^2 E_x}{\partial t^2}$$

$$\frac{\partial^2 E_y}{\partial y^2} = \mu\epsilon \frac{\partial^2 E_y}{\partial t^2}$$

$$\frac{\partial^2 E_z}{\partial z^2} = \mu\epsilon \frac{\partial^2 E_z}{\partial t^2}$$

We have

$$\nabla.\text{D} = \dot{\rho}_v$$

$$\nabla.\text{E} = \frac{\dot{\rho}_v}{\epsilon}$$

$$\nabla.\text{E} = 0 \text{ Since } \rho_v = 0$$

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0 \dots\dots\dots\dots\dots\dots.(2)$$

The wave is independent of y,z and is function of x and t only

$$\therefore \qquad \frac{\partial E_y}{\partial y} = 0; \qquad \frac{\partial E_z}{\partial z} = 0$$

By substituting these in above equation

$$\frac{\partial E_x}{\partial x} = 0$$

Differentiating with respect to the x for above equation, we get

$$\frac{\partial^2 E_x}{\partial x^2} = 0$$

Substituting this in wave equation

$$\mu\epsilon \frac{\partial^2 E_x}{\partial x^2} = 0$$

$\mu \neq 0, \epsilon \neq 0$This an partial differential equation

The possible solution is $E_x = 0$

The physical explanation of this is the wave travelling in the x-direction does not have a component of the field E in the direction of propagation.

Similarly the wave does not have a component of H along in the x-direction.

This proves that the wave is propagating in the x-direction, and has the components of E and H in y and z directions only, which are normal to the direction of propagation of the wave.

Thus the uniform plane wave is referred to as transverse Electromagnetic wave (TEM).

MUTUALLY PERPEDICULAR RELATION'S BETWEEN E AND H FOR UNIFORM PLANE WAVES:

Form the above equation Consider a uniform plane wave travelling along the x-direction.

We know that along the direction of propa01gation E and H components are zero.

i.e. E_x=0, H_x=0

Consider $\nabla \times E = -\frac{\partial B}{\partial t}$

$$\begin{vmatrix} i_x & i_y & i_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_y & E_z \end{vmatrix} = -\frac{\partial}{\partial t}(B_x i_x + B_y i_y + B_z i_z)$$

For a uniform plane wave propagating in the x-direction, the wave is independent of y, z and is function of x and t only.

$$\begin{vmatrix} i_x & i_y & i_z \\ \frac{\partial}{\partial x} & 0 & 0 \\ 0 & E_y & E_z \end{vmatrix} = -\frac{\partial}{\partial t}(B_x i_x + B_y i_y + B_z i_z); \quad B_x = 0$$

$$-i_y \frac{\partial E_z}{\partial x} + i_z \frac{\partial E_y}{\partial x} = \frac{\partial B_x}{\partial t} i_x - \frac{\partial B_y}{\partial t} i_y - \frac{\partial B_z}{\partial t} i_z$$

$$\frac{\partial E_z}{\partial x} = \frac{\partial B_y}{\partial t} = \mu \frac{\partial H_y}{\partial t} \dots\dots\dots\dots (1)$$

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} = -\mu \frac{\partial H_z}{\partial t} \dots\dots (2)$$

Consider the Maxwell's equation

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

In free space J=0

$$\nabla \times H = \frac{\partial D}{\partial t}$$

$$\begin{vmatrix} i_x & i_y & i_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = \frac{\partial}{\partial t}(D_x i_x + D_y i_y + D_z i_z)$$

Wave is propagating along x-direction

$$\frac{\partial}{\partial y} = 0 \; ; \; \frac{\partial}{\partial z} = 0 \; H_x = 0, D_x = 0$$

$$\begin{vmatrix} i_x & i_y & i_z \\ \dfrac{\partial}{\partial x} & 0 & 0 \\ 0 & H_y & H_z \end{vmatrix} = \frac{\partial}{\partial t}(D_y i_y + D_z i_z)$$

$$-i_y \frac{\partial H_z}{\partial x} + i_z \frac{\partial H_y}{\partial x} = \frac{\partial D_y}{\partial t} i_y + \frac{\partial D_z}{\partial t} i_z$$

From the above equation.

$$\frac{\partial H_z}{\partial x} = -\frac{\partial D_y}{\partial t} = \in \frac{\partial E_y}{\partial t} \dots \dots \dots \dots (3)$$

$$\frac{\partial H_y}{\partial x} = \frac{\partial D_z}{\partial t} = \in \frac{\partial E_z}{\partial t} \dots \dots \dots \dots .. (4)$$

Let us consider the wave

$$E_y = E_0 \cos(wt - \beta X)$$

Differentiating with respect to ‘x’

$$\frac{\partial E_y}{\partial x} = E_0 \beta \sin\,(wt - \beta X)$$

Substituting this in equation(2)

$$-\mu \frac{\partial H_z}{\partial x} = E_0 \beta \sin\,(wt - \beta X)$$

$$\frac{\partial H_z}{\partial x} = -\frac{E_0 \beta}{\mu} \sin(wt - \beta X)$$

Integrating on both sides with respect to‘t’

$$H_z = -\frac{E_0 \beta}{\mu} \int \sin(wt - \beta X) + c$$

$$H_z = -\frac{E_0 \beta}{\mu} - \frac{\cos\,(wt - \beta X)}{\mathrm{w}} + \mathrm{c}$$

$$H_z = \frac{\beta}{w\mu}\; E_0 \cos(wt - \beta X) + c$$

$$H_z = \frac{\beta}{w\mu} E_y + c$$

By neglecting constant ‘C’ which is independent of‘t’

$$H_z = \frac{\beta}{w\mu} E_y$$

$$\frac{E_y}{H_z} = \frac{w\mu_0}{w\sqrt{\mu_0 \epsilon_o}}$$

$$\frac{E_y}{H_z} = \sqrt{\frac{\mu_0}{\epsilon_o}} \dots \dots (5)$$

Similarly if

$$E_z = E_0 \cos\,(wt - \beta X)$$

$$\frac{\partial E_z}{\partial x} = E_0 \beta \sin\,(wt - \beta X)$$

Substituting this in equation (1)

$$\mu \frac{\partial H_z}{\partial x} = E_0 \beta \sin\,(wt - \beta X)$$

$$\frac{\partial H_z}{\partial x} = \frac{E_0 \beta}{\mu} \sin(wt - \beta X)$$

Integrating on both sides with respect to‘t’

$$H_z = \frac{\beta}{w\mu}\; E_0 \cos(wt - \beta X) + c$$

By neglecting constant ‘C’ which is independent of‘t’

$$H_z = -\frac{\beta}{w\mu} E_y$$

$$\frac{E_z}{H_y} = \frac{w\mu_0}{\beta}$$

$$if\ \beta = w\sqrt{\mu\epsilon}$$

$$\frac{E_z}{H_y} = -\sqrt{\frac{\mu}{\in}}$$

For free space

$$\frac{E_z}{H_y} = -\sqrt{\frac{\mu_0}{\epsilon_o}}$$

Since $E = \sqrt{E_x^2 + E_x^2}$ and $H = \sqrt{H_x^2 + H_x^2}$

Where E and H are the total electric and magnetic field strength's then

$$\frac{E}{H} = \sqrt{\frac{\mu_0}{\epsilon_o}}$$

Since the units of E are V/m and the units of H are A/m, the ratio E/H will have the dimensions of impedance or ohms.

For this reason the ratio

$$\frac{E}{H} = \sqrt{\frac{\mu}{\epsilon}}$$

is called as "characteristic impedance" or "intrinsic impedance" of the medium.

INTRINSIC IMPEDANCE:-

The ratio of amplitudes of electric field intensity to the magnetic field intensity is known as "intrinsic impedance" of the medium in which the wave is travelling, it is denoted by η

$$\frac{E}{H} = \eta = \sqrt{\frac{\mu}{\epsilon}}$$

The intrinsic impedance for free space is given by

$$\frac{E}{H} = \sqrt{\frac{\mu_0}{\epsilon_o}} = 120\pi = 377\Omega$$

The dot product of E and H gives. E.H= Ey Hy+ Ez Hz

$$E.H = \eta\, Hx\, Hy - \eta Hy\, Hz = 0$$

Thus in a uniform plane wave, E and H are at right angles to each other.

The cross product of E and H gives

$$E \times H = \begin{vmatrix} i_x & i_y & i_z \\ 0 & E_y & E_Z \\ 0 & H_y & H_z \end{vmatrix}$$

$$= i_x[E_y H_z - E_Z H_y]$$

$$E \times H = \left[\eta H_Z^2 - \eta H_Y^2\right] = \eta H^2$$

Thus the electric field vector crossed in to the magnetic field vector gives the direction in which the wave travels.

<u>CHARACTERISTICS OF UNIFORM PLANE WAVES:-</u>

1. Uniform plane wave is a TEM wave.
2. The E and H field's are perpendicular to the direction of propagation.
3. E.H=0 i.e E and H are at right angles to each other.
4. E x H determines the direction of propagation.

5. The intrinsic impedance of uniform plane wave for free space is given by 120 π = 377Ω.

Wave propagation in conducting medium

Wave equations for electric and magnetic fields are

$$\nabla^2 E = \mu\sigma \frac{\partial E}{\partial t} + \mu\varepsilon \frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 H = \mu\sigma \frac{\partial H}{\partial t} + \mu\varepsilon \frac{\partial^2 H}{\partial t^2}$$

Wave equations for electric and magnetic fields in phasor form are:

$$\nabla^2 E = j\omega\mu(\sigma + j\omega\varepsilon)E$$

$$\nabla^2 H = j\omega\mu(\sigma + j\omega\varepsilon)H$$

Consider

$$\nabla^2 E = j\omega\mu(\sigma + j\omega\varepsilon)E$$

$$\nabla^2 E = \gamma^2 E$$

Where γ is known as propagation constant and is defined by the equation

$$\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$

The propagation constant 'γ' is a complex number having real and imaginary parts designated by 'α' and 'β' respectively, that is

$$\gamma = \alpha + j\beta$$

The real part ‘α’ is known as attenuation constant and its units are Nep/meter.

The imaginary part is denoted by ‘β’ is known as phase constant and its units are rad/meter.

Consider the equation

$$\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$$

$$\gamma^2 = -\omega^2\mu\varepsilon + j\omega\mu\sigma$$

$$\gamma = \alpha + j\beta$$

$$\gamma^2 = \alpha^2 - \beta^2 + 2j\alpha\beta$$

$$\alpha^2 - \beta^2 = -\omega^2\mu\varepsilon$$

$$2\alpha\beta = \omega\mu\sigma$$

$$\gamma^2 = \alpha^2 + \beta^2 = \sqrt{(-\omega^2\mu\varepsilon)^2 + (\omega\mu\sigma)^2}$$

$$= \sqrt{\omega^4\mu^2\varepsilon^2 + \omega^2\mu^2\sigma^2}$$

$$\alpha^2 + \beta^2 = \omega^2\mu\varepsilon\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} \longrightarrow \text{(a)}$$

$$\alpha^2 - \beta^2 = -\omega^2\mu\varepsilon \longrightarrow \text{(b)}$$

Adding equations (a) and (b) we get

$$2\alpha^2 = \omega^2\mu\varepsilon\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} - \omega^2\mu\varepsilon$$

$$\alpha^2 = \frac{\omega^2\mu\varepsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]$$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}\right)-1}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right)}$$

Relations between *E* and *H* for uniform plane waves are

$$\frac{\partial E_z}{\partial x} = \mu\frac{\partial H_y}{\partial t} \longrightarrow \quad (I)$$

$$\frac{\partial E_y}{\partial x} = -\mu\frac{\partial H_z}{\partial t} \longrightarrow \quad (II)$$

Let a wave travelling in the positive *x* direction have a *z* component denoted as

$$E_z = E_o\, e^{j\omega t}\, e^{-\gamma x}$$

Differentiating with respect to '*x*'

$$\frac{\partial E_z}{\partial x} = E_o\, e^{j\omega t}\, e^{-\gamma x}(-\gamma)$$

Substituting this in equation (I)

$$\mu\frac{\partial H_y}{\partial t} = -\gamma\, E_o\, e^{j\omega t}\, e^{-\gamma x}$$

$$\frac{\partial H_y}{\partial t} = -\frac{\gamma E_o}{\mu}\, e^{j\omega t}\, e^{-\gamma x}$$

Integrating with respect to '*t*' on both sides

$$H_y = -\frac{\gamma E_o}{\mu}\, e^{-\gamma x}\frac{e^{j\omega t}}{j\omega} + C$$

By neglecting the integrating constant

$$H_y = \frac{-\gamma}{j\omega\mu} E_z$$

$$\frac{E_z}{H_Y} = \frac{-j\omega\mu}{\gamma}$$

We know that $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$

$$\therefore \quad \frac{E_z}{H_y} = -\frac{j\omega\mu}{\sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}}$$

$$\frac{E_Z}{H_y} = -\sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} \longrightarrow \text{(III)}$$

$$\frac{E_y}{H_Z} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} \longrightarrow \text{(IV)}$$

From equation (III)

$$E_z{}^2 = \frac{j\omega\mu}{\sigma + j\omega\varepsilon} H_y{}^2$$

From equation (IV)

$$E_y{}^2 = \frac{j\omega\mu}{\sigma + j\omega\varepsilon} H_z{}^2$$

$$|E| = \sqrt{E_z{}^2 + E_y{}^2}$$

$$|E| = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}\left(\sqrt{H_y{}^2 + H_z{}^2}\right)$$

$$|E| = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} \cdot |H|$$

$$\left|\frac{E}{H}\right| = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \eta = intinsic\ impedance\ of\ the\ medium$$

Frequency:

The number of oscillations carried out by a wave per second is known as frequency of the wave .it is denoted by '*f*' and the units are hertz (Hz).

The angular frequency will be given by $\omega = 2\pi f$ and the units are *rad/sec*.

Wavelength:

Wavelength is defined as the distance that a wave travels along the line in which its phase angle changes by 2π radians. It is denoted by 'λ' and the units are *'meters'*.

$$i.e. \qquad \beta \cdot \lambda = 2\pi \qquad \Rightarrow \qquad \lambda = \frac{2\pi}{\beta}$$

$$\therefore \qquad \beta = \frac{2\pi}{\lambda}$$

Velocity:

We know that $\vartheta = f * \lambda$

$$\vartheta = \frac{\omega}{2\pi} * \frac{2\pi}{\beta}$$

$$\vartheta = \frac{\omega}{\beta}\ m/sec$$

$$\beta = \omega\sqrt{\mu\varepsilon} = \frac{2\pi}{\lambda}$$

$$\beta = \frac{\omega}{\vartheta}$$

$$= \frac{2\pi f}{f * \lambda}$$

$$= \frac{2\pi}{\lambda}$$

In free space the velocity of wave propagation is equal to the velocity of light

$C = 3 * 10^8 m/s$

Conductors and dielectrics

In Electromagnetics, materials are divided into two classes:

1. Conductors
2. Dielectrics (or) insulators

From Maxwell's first equation

$$\nabla \ X \ H = \sigma E + j\omega\varepsilon E$$

the ratio $\frac{\sigma}{\varepsilon\omega}$ is the ratio of conduction current density to displacement current density. Hence $\frac{\sigma}{\varepsilon\omega} = 1$ can be considered to mark the dividing line between conductors and dielectrics.

- For good conductors such as metals $\frac{\sigma}{\varepsilon\omega}$ is very much greater than unity over the entire radio frequency spectrum.
- For good dielectric or insulators $\frac{\sigma}{\varepsilon\omega}$ is very much less than unity in the radio frequency range.
- For good conductors σ and ε are nearly independent of frequency.

- For dielectrics the constants σ and ε are functions of frequency, but the ratio $\frac{\sigma}{\varepsilon\omega}$ is often relatively constant over the frequency range of interest.
- The ratio $\frac{\sigma}{\varepsilon\omega}$ is known as the "dissipation factor" of the dielectric for small values of dissipation factor, is same as the power factor.
- The displacement current density leads conduction current density by 90^0

$$\tan\theta = \left|\frac{J_\sigma}{J_D}\right| = \frac{\sigma}{\varepsilon\omega}$$

Where $\tan\theta$ is the loss tangent $'\theta'$ is the loss angle of the medium.

- A medium is said to be good dielectric if $\tan\theta$ is very small $\sigma \ll \omega\varepsilon$.
- A medium is said to be good conductor if $\tan\theta$ is very large $\sigma \gg \omega\varepsilon$.

∴ Small the angle, lower is the power loss in the medium.

Classification of Media:

Free space	$\sigma = 0$	$\varepsilon = \varepsilon_o$	$\mu = \mu_o$
Loss less dielectrics	$\sigma = 0$	$\varepsilon = \varepsilon_o \varepsilon_r$	$\mu = \mu_o \mu_r$
Lossy dielectrics	$\sigma \neq 0$	$\varepsilon = \varepsilon_o \varepsilon_r$	$\mu = \mu_o \mu_r$
Good conductors	$\sigma \cong \infty$	$\varepsilon = \varepsilon_o$	$\mu = \mu_o \mu_r$

Wave motion in free space: The free space is characterized by

$\sigma = 0, \mu = \mu_o, \epsilon = \epsilon_o$

The attenuation constant:

$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2}\right) - 1}$$

For free space

$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}}[1 - 1] = 0$$

$$\therefore \alpha = 0$$

Phase constant:

$$\beta = \omega \sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right)}$$

For free space

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{\not{2}}[\not{2}]}$$

$$\therefore \beta = \omega\sqrt{\mu\varepsilon}$$

$$\therefore \beta = \omega\sqrt{\mu_o\varepsilon_o}$$

Intrinsic impedance:

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

$$\eta = \sqrt{\frac{j\omega\mu}{j\omega\varepsilon}}$$

$$\eta = \sqrt{\frac{\mu_o}{\varepsilon_o}} = 120\pi = 377\Omega$$

Phase velocity:

$$\vartheta = \frac{\omega}{\beta}$$

$$\vartheta = \frac{\omega}{\omega\sqrt{\mu_o\varepsilon_o}}$$

$$\vartheta = \frac{1}{\sqrt{\mu_o\varepsilon_o}} = 3 * 10^8 m/sec = C$$

Hence the electromagnetic wave travels with the velocity of light in free space.

Wave equation for electric field in free space:

$$\nabla^2 E = \sigma\mu\frac{\partial E}{\partial t} + \mu\varepsilon\frac{\partial^2 E}{\partial t^2}$$

For free space $\sigma = 0$

$$\nabla^2 E = \mu_o \varepsilon_o \frac{\partial^2 E}{\partial t^2}$$

Wave equation for magnetic field in free space:

$$\nabla^2 H = \sigma\mu \frac{\partial H}{\partial t} + \mu\varepsilon \frac{\partial^2 H}{\partial t^2}$$

For free space $\sigma = 0$

$$\nabla^2 H = \mu_o \varepsilon_o \frac{\partial^2 H}{\partial t^2}$$

Wave equation in phasor form is given by

$$\nabla^2 E = -\omega^2 \mu_o \varepsilon_o E \; ; \; \nabla^2 H = -\omega^2 \mu_o \varepsilon_o H$$

Wave propagation in good dielectrics:

For good dielectrics $\frac{\sigma}{\omega\varepsilon} \ll 1$

$$\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} \cong 1 + \frac{1}{2}\left(\frac{\sigma}{\omega\varepsilon}\right)^2$$

this is good approximation for above condition by taking the first two terms of the binomial expansion .

The attenuation constant 'α' is given by

$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2}\right) - 1}$$

$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\left(1 + \frac{\sigma^2}{2\omega^2\varepsilon^2} - 1\right)}$$

$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\left(\frac{\sigma^2}{2\omega^2\varepsilon^2}\right)}$$

$$\therefore \boxed{\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}}$$

The expression for phase constant 'β' is

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right)}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(1+\frac{1}{2}\left(\frac{\sigma}{\omega\varepsilon}\right)^2+1\right)}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(1+\frac{1}{2}\left(\frac{\sigma}{\omega\varepsilon}\right)^2+1\right)}$$

$$\beta = \omega\sqrt{\mu\varepsilon}\sqrt{\left(1+\frac{\sigma^2}{4\omega^2\varepsilon^2}\right)}$$

$$\therefore \boxed{\beta = \omega\sqrt{\mu\varepsilon}\left(1+\frac{\sigma^2}{8\omega^2\varepsilon^2}\right)}$$

The velocity of the wave in the dielectric is given by

$$\vartheta = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu\varepsilon}\left(1+\frac{\sigma^2}{8\omega^2\varepsilon^2}\right)}$$

$$\therefore \boxed{\vartheta = \frac{1}{\sqrt{\mu\varepsilon}\left(1+\frac{\sigma^2}{8\omega^2\varepsilon^2}\right)}}$$

The intrinsic impedance is given by

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

$$\eta = \sqrt{\frac{j\omega\mu}{j\omega\varepsilon}\left(\frac{1}{1 + \frac{\sigma}{j\omega\varepsilon}}\right)}$$

$$\therefore \boxed{\eta = \sqrt{\frac{\mu}{\varepsilon}}\left[1 + \frac{j\sigma}{2\omega\varepsilon}\right]}$$

Wave motion in lossy dielectric:

A lossy dielectric is characterized by $\sigma = 0, \mu = \mu_o\mu_r, \varepsilon = \varepsilon_o\varepsilon_r$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2}\right) - 1}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right)}$$

$$\lambda = \frac{2\pi}{\beta}$$

$$\vartheta = \frac{\omega}{\beta}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

From Maxwell's equation we know that

$$\nabla \times H = J + J_D = J + \frac{\partial D}{\partial t}$$

In phasor form

$$\nabla \times H = J + j\omega\varepsilon E = \sigma E + j\omega\varepsilon E$$

$$\left|\frac{J_C}{J_D}\right| = \frac{\sigma}{\omega\varepsilon} = \tan\theta$$

Where $\tan\theta$ is the loss tangent.

A medium is said to be good dielectric if $\tan\theta$ is very small $\sigma \ll \omega\varepsilon$

$\therefore$ Small the angle θ,lower is the power loss in the medium.

Wave motion in good conductors:

A good conductor is characterized by high $\sigma, \mu = \mu_o\mu_r, \varepsilon = \varepsilon_o$ and $\sigma \gg \omega\varepsilon$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$

$$\gamma = \sqrt{j\omega\mu\sigma\left(1 + \frac{j\omega\varepsilon}{\sigma}\right)}$$

$$\frac{\sigma}{\omega\varepsilon} \gg 1 \qquad\qquad \therefore \frac{\omega\varepsilon}{\sigma} \ll 1$$

$$\therefore \gamma = \sqrt{j\omega\mu\sigma}$$

$$\gamma = \sqrt{\omega\mu\sigma}\,\angle 45^0$$

$$\gamma = \sqrt{\omega\mu\sigma}\;[\cos 45 + j\sin 45]$$

$$\gamma = \sqrt{\omega\mu\sigma}\;\left[\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right]$$

$$\gamma = \sqrt{\omega\mu\sigma}$$

$$\gamma = \sqrt{\frac{\omega\mu\sigma}{2}} + j\sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\gamma = \alpha + j\beta$$

$$\therefore \alpha = \sqrt{\frac{\omega\mu\sigma}{2}}, \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f \mu\sigma}$$

The velocity of propagation

$$\vartheta = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\frac{\omega\mu\sigma}{2}}} = \sqrt{\frac{2\omega}{\mu\sigma}}$$

The intrinsic impedance of the conductor is given by

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma\left(1 + \frac{j\omega\varepsilon}{\sigma}\right)}}$$

$$\eta \cong \sqrt{\frac{j\omega\mu}{\sigma}}$$

$$\eta \cong \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^0$$

In a conductor α and β are large, the wave attenuates greatly as it progress and phase shift per unit length is also large. The velocity of the wave is small, the intrinsic impedance is small and has a reactive component. The angle of this impedance is always 45^0 for good conductors.

PROBLEMS:

In a medium $E = 16e^{-x/20}\sin(2*10^8 t - 2x)i_z \;\; v/m$. Find the direction of propagation, the propagation constant, wave length, speed of the wave and skin depth.

Sol: Given $E = 16e^{-x/20}\sin(2*10^8 t - 2x)i_z \;\; v/m$ comparing the above equation with $E = E_o e^{-\alpha x}\sin(\omega t - \beta x)i_z$ we have

$$\alpha = \frac{1}{20} = 0.05\;;\omega = 2*10^8\frac{rad}{sec};\;\beta = 2$$

The above equation indicates that the wave is propagating in positive *x* direction.

The propagation constant $\gamma = \alpha + j\beta$

$$\therefore \quad \gamma = 0.05 + j2$$

Wave length

$$\lambda = \frac{2\pi}{\beta}$$

$$\lambda = \frac{2\pi}{2} = \pi\; mt = 3.14\; mt$$

Speed of the wave

$$\vartheta = \frac{\omega}{\beta}$$

$$\vartheta = \frac{2 * 10^8}{2} = 10^8 mt/sec$$

Skin depth

$$\delta = \frac{1}{\alpha}$$

$$= \frac{1}{0.05}$$

$$\delta = 20mt$$

2. If *H* field of EM wave is given by $H(z,t) = 48\cos(10^8 t + 40z) i_y Amp/mt$. Find out the wavelength in free space.

Sol: Given $H(z,t) = 48\cos(10^8 t + 40z) i_y Amp/mt$

$$\lambda = \frac{2\pi}{\beta}$$

$$\beta = 40$$

$$\lambda = \frac{2\pi}{40}$$

$$= \frac{\pi}{20} mt$$

Ex: If ε_r=9, $\mu=\mu_0$ for the medium in which a wave with frequency $f = 0.3GHz$ is propagating, determine propagation constant and intrinsic impedance of the medium When i) σ =0 and ii) σ =10mho/m.

Sol: given ε_r=9, $\mu=\mu_0$, $f = 0.3GHz = 0.3 \times 10^9$Hz

γ =? η =?

σ =0

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right)}$$

For σ=0 α=0

$$\beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}$$

After subsisting all values

β= 18.84 rad/m

Then γ=α+jβ

γ=j18.84rad/m.

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma+j\omega\varepsilon}}$$

For σ=0; $\eta = \sqrt{\frac{\mu}{\varepsilon}}$

After subsisting μ, ε values

η=125.66 ohms.

σ =10mho/m

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right)}$$

After subsisting all values

α=108.01 nepers/m

β=109.64 rad/m

Then γ=α+jβ=108.01+109.64

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma+j\omega\varepsilon}}$$

η=4.866angle (44.57^0) ohms.

Ex: A uniform plane wave with E=$E_x\ i_x$ propagates in a lossless simple medium (ε_r=4, $\mu_r = 1, \sigma = 0$) in the +z direction .assume that E_x is sinusoidal with a frequency of 100 MHz and has a maximum value of 10^{-4}v/m at t=0 and z=1/8m.

write the instantaneous expression for E for any t and z

Write the instantaneous expression for H.

Sol: given data ε_r=4, $\mu_r = 1 , \sigma = 0$

Wave propagates in the +z direction

f=100 MHz=10^8Hz

$$E = E_0 e^{-\alpha z} sin(\omega t - \beta z) i_x$$

$$\sigma = 0, \alpha = 0.$$

$$\beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}$$

$$\beta = 1.33\Pi\ rad/m$$

$$E = 10^{-4} sin\ (2\Pi\times 10^8 t - 1.33\Pi z)\ i_x\ v/m$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}} = 60\Pi$$

$$H = \frac{E}{\eta} = \frac{10^{-4}}{60\Pi} sin\ (2\Pi\times 10^8 t - 1.33\Pi z)\ i_x A/m$$

Skin depth (or) depth of penetration:

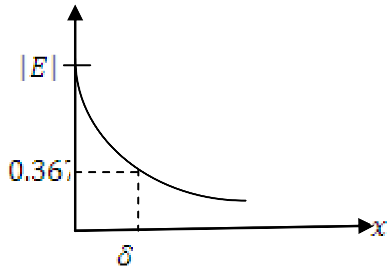
In a medium which has high conductivity the wave is attenuated as it progresses due to the losses.

In a good conductor at radio frequencies the rate of attenuation is very great and the wave may penetrate only a very short distance.

The depth of penetration 'δ' is defined as the depth in which the wave has been attenuated to $\frac{1}{e}$ (or) approximately 37% of the original value

The wave in a conducting medium may in general represented as

E=$E_0 e^{-\alpha x}$



At x=0 E=E_0

At x=$\frac{1}{\alpha}$ E=$E_0 e^{-1}$

E=0.37678 E_0=37% E_0

αx=1 (or)

αδ=1

$$\delta = \frac{1}{\alpha} = \frac{1}{\omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right)}}$$

For a good conductor $\frac{\sigma}{\omega\varepsilon}$>>1, so

$$\delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\frac{\omega\sigma\mu}{2}}} = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{1}{\Pi f\mu\sigma}} = \frac{1}{\beta}$$

The phenomenon where by field intensity in a conductor rapidly decreases is known as "skin effect".

Surface Impedance:

At high frequency the current is confined almost entirely to a very thin sheet at the surface of the conductor.

The surface impedance defined by

$$Z_s = \frac{E_{tan}}{J_s}$$

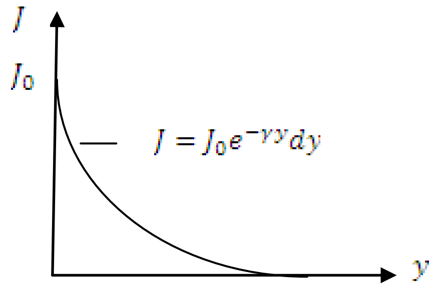
Where E_{tan} is the electric field strength parallel to the surface of the conductor.

J_s is the linear current density represents the total conduction current per meter width, flowing in the thin sheet.

If it is assumed that the conductor is a flat plate with its surface at the y=0 plane the current distribution in the y-direction is given by

$$J = J_0 e^{-\gamma y}$$

Where J_0 is given the current density at the surface.



It is assumed that the thickness of the conductor is very much greater than the depth of penetration, so that there is no reflection from the back surface of the conductor.

The total conduction current per meter width,

i.e. the linear current density is

$$J_s = \int_0^{\alpha} J dy = \int_0^{\alpha} J_0 \, e^{-\gamma y} dy$$

$$J_s = J_0 \frac{e^{-\gamma y}}{-\gamma} dy$$

$$J_s = \frac{-J_0}{\gamma} [0 - 1] = \frac{J_0}{\gamma}$$

But the current density at the surface is

$$J_0 = \sigma E_{tan}$$

$$Z_s = \frac{E_{tan}}{J_s} = \frac{E_{tan}}{\frac{J_0}{\gamma}}$$

$$Z_s = \frac{E_{tan}\gamma}{J_0}$$

$$Z_s = \frac{E_{tan}\gamma}{\sigma E_{tan}}$$

$$Z_s = \frac{\gamma}{\sigma}$$

The propagation constant 'γ' in a conducting medium was found to be

$$\gamma = \sqrt{j\omega\omega(\sigma + j\omega\varepsilon)}$$

$$\gamma = \sqrt{j\omega\mu\sigma}$$

for a thick conductor

$$Z_s = \frac{\gamma}{\sigma} = \frac{\sqrt{j\omega\mu\sigma}}{\sigma}$$

$$Z_s = \sqrt{\frac{j\omega\mu}{\sigma}} = \eta \text{ (for the conducting medium)}$$

The intrinsic impedance of good conductor is $\eta = \sqrt{\frac{j\omega\mu}{\sigma}}$

"There fore for good conductors the surface impedance of a plane conductor that is very much thicker than the skin depth is just equal to the characteristic impedance of the conductor"

Skin Effect Resistance:

If the conductor thickness is very much greater than the depth of penetration the surface impedance of such a conductor is

$$Z_s = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}} \text{ angle of } 45^0$$

$$Z_s = \sqrt{\frac{\omega\mu}{2\sigma}} + j\sqrt{\frac{\omega\mu}{2\sigma}}$$

The real part of the surface impedance is the high frequency (or) skin effect resistance per unit length of a flat conductor of unit width

$$R_s = \sqrt{\frac{\omega\mu}{2\sigma}}$$

The skin depth in a conductor is

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}}$$

$$R_s = \frac{1}{\sigma\delta}$$

The surface resistance of a flat conductor at any frequency is equal to the d.c resistance of a thickness δ of the same conductor.

The power loss per unit area of plane conductor will be given by $J_{s\,eff}^2 R_s$

Where J_s liner current density (or) current per meter width (effective value)

Note:

The electric and magnetic fields for free space are represented by the equations

$$E = E_0 e^{-\alpha x} \cos(\omega t - \beta x)$$

$$H = \frac{E_0}{\eta} e^{-\alpha x} \cos(\omega t - \beta x - \theta y)$$

The electric and magnetic fields for good conductors are represented by the following equations

$$E = E_0 e^{-\alpha x} \cos(\omega t - \beta x)$$

$$H = \frac{E_0}{\eta} e^{-\alpha x} \cos(\omega t - \beta x - \frac{\pi}{4})$$

Ex: Explain the term "skin effect resistance and obtain its value for copper at on 1MHz Assume the conductivity of copper as 5.8×10^7 mhos/meter [$\mu=\mu_0$]

Sol: given

f = 1 MHz= 10^6Hz

σ= 5.8× 10^7mhos/meter

μ=μ_0= 4Π× 10^{-7} H/m

Skin effect resistance $R_s = \sqrt{\frac{\omega\mu}{2\sigma}}$

$$R_s = \sqrt{\frac{\Pi f \mu}{\sigma}}$$

$$R_s = 0.26\times 10^{-3} \text{ohms.}$$

Ex: If distilled water constants $\mu_r = 1, \varepsilon_r = 81$ and power factor 0.05 at 1 GHz calculate the depth of penetration.

Sol: given $\mu_r = 1$

$\varepsilon_r = 81$

Power factor $=\frac{\sigma}{\omega\varepsilon}$=0.05

f= 1GHz=10^9 Hz

δ=?

σ= ωε0.05

σ= 0.225mho/meter

$$\delta=\sqrt{\frac{2}{\omega\mu\sigma}}$$

$$\delta=\sqrt{\frac{1}{\Pi f \mu\sigma}}$$

$$\delta=3.55\times 10^{-3}\text{m}$$

Polarization :

Polarization refers to the physical orientation of the radiated electromagnetic waves in space.

(Or)

The polarization of a uniform plane wave refers to the time varying behavior of the electric field strength vector at some fixed point in space

Consider for example a uniform plane wave traveling in the Z-direction with E and H vectors lying in the x-y plane.

If $E_y = 0$ and only E_x is present the wave is said to be polarized in the x-direction.

If $E_x = 0$ and only E_y is present the wave is said to be polarized in the y-direction.

If both E_x and E_y are present are in phase the resultant electric field has a direction dependent on the relative magnitude of E_x and E_y. If the direction of resultant vector is constant with time, and it passes through a straight line then the wave is said to be "Linearly Polarized".

If E_x and E_y are not in phase that is if they reach their maximum values at different instants of time ,then the direction of the resultant electric vector will vary with time.

If E has E_x and E_y components which are having equal magnitudes and 90^0 phase difference, then the locus of resultant E will from a circle over a period of time. Such a wave called as "circularly polarized".

If E has E_x and E_y components having different magnitudes and 90^0 phase difference, then the locus of resultant E will be "elliptically polarized".

For positive Z-propagation, the wave would generally have its electric field pharos expressed

$E = (E_{x0}i_x + E_{y0}i_y)e^{-\alpha z}.e^{-j\beta z}$

If Φ is the phase difference between E_{x0} and E_{y0} where Φ<$\frac{\Pi}{2}$ and considering the propagation in a lossless medium, then the total field in phasor from is

$E = (E_{x0}i_x + E_{y0}i_y e^{j\Phi})e^{-j\beta z}$

By multiplying with $e^{j\omega t}$ for getting time varying field and taking the real part

$E\ (z,t) = E_{x0}\cos(\omega t - \beta z)\, i_x + E_{y0}\cos\ (\omega t - \beta z + \Phi) i_y$

Elliptical polarization occurs when $E_{x0} \neq E_{y0}$ and when Φ=± $\frac{\Pi}{2}$

$E(z,t) = E_{x0}\cos(\omega t - \beta z)\, i_x + E_{y0}\cos\ (\omega t - \beta z \pm \frac{\Pi}{2}) i_y$

$E(z,t) = E_{x0}\cos(\omega t - \beta z)\, i_x \pm E_{y0}\sin(\omega t - \beta z) i_y$

Circular polarization occurs when $E_{x0} = E_{yo} = E_0$ and when Φ=± $\frac{\Pi}{2}$

Then E(z,t)=$E_0[\cos(\omega t - \beta z)\, i_x + \cos\left(\omega t - \beta z \pm \frac{\Pi}{2}\right) i_y]$

$E(z,t) = E_0[\cos(\omega t - \beta z)\, i_x \pm \sin(\omega t - \beta z)\, i_y]$

If we consider a fixed position along z-axis (such as z=0) and allow time to vary with Φ=+ $\frac{\Pi}{2}$ the above equation becomes

$$E(0,t) = E_0[\cos(\omega t)\, i_x - \sin(\omega t)\, i_y] \qquad 1$$

Choosing Φ=+$\frac{\pi}{2}$ leads to the field vector rotates in the clock wise direction.

The wave exhibits left circular polarization, with left hand thumb in the direction of propagation the fingers curl in the field rotation direction. Thus with forward z-propagation, equation 1 describes a "Left circularly polarized wave".

If $\Phi = -\frac{\pi}{2}$

$$E\,(0,t) = E_0[\cos(\omega t)\, i_x + \sin(\omega t)\, i_y] \qquad 2$$

Choosing Φ= - $\frac{\pi}{2}$ leads to the field vector rotates in the counter- clock wise direction.

Thus with forward z-propagation, equation 2 describes a "Right circularly polarized wave".

Let the electric field components

$$E_x = E_1 \sin(\omega t - \beta z)$$

$$E_y = E_2 \sin(\omega t - \beta z + \delta)$$

If E_1=E_2=E_0 and δ =$\pm 90^0$ the wave is circularly polarized.

$E = E_0[\sin(\omega t - \beta z)\, i_x + \cos(\omega t - \beta z \pm 90^0)\, i_y]$

When$\delta = +90^0$, the wave is left circularly polarized.

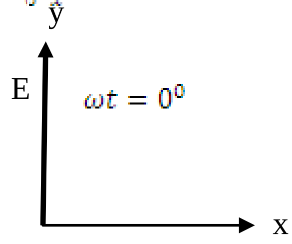
For the case$\delta = -90^0$, the wave is right circularly polarized.

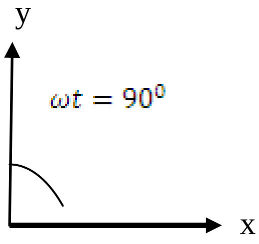
For the case$\delta = +90^0$, and for z=0 for t=0

$E = E_0 i_y$

After one quarter cycle later ωt=90^0

$E = E_0 i_x$





Thus at fixed position the electric filed vector rotates clock wise .according to the IEEE definition this corresponds to the left circular polarization.

The opposite rotation direction $\delta=-90^0$ corresponds to right circular polarization.

Q): The electric filed of an electromagnetic wave propagating in the positive z- direction is given by E=$\sin(\omega t - \beta z)\, i_x + \cos(\omega t - \beta z \pm 90^0)\, i_y$ the wave is

A) Linearly polarized in the z-direction

B) Elliptically polarized

c) Left –hand circularly polarized

D) Right-hand circularly polarized

Ans: “c”

Poynting theorem:

Energy can be transported from one point to another point by means of EM waves. The rate of such energy transportation can be obtained from Maxwell's equations;

$$\nabla \times E = -\frac{\partial B}{\partial t} = -\mu \frac{\partial H}{\partial t}$$

$$\nabla \times H = J + \frac{\partial D}{\partial t} = \sigma E + \epsilon \frac{\partial E}{\partial t}$$

Taking dot product with E on both sides of the above equation gives,

$$E.(\nabla \times H) = E.(\sigma E) = E.\epsilon \frac{\partial E}{\partial t}$$

By using vector identity,

$$\nabla.(A \times B) = B.(\nabla \times A) - A.(\nabla \times B)$$

$$\nabla.(E \times H) = H.(\nabla \times E) - E.(\nabla \times H)$$

$$\mathrm{E}.(\nabla \times H) = H.(\nabla \times E) - \nabla.(\mathrm{E} \times H)$$

Substituting this equations in above equation,

$$H.(\nabla \times E) - \nabla.(\mathrm{E} \times H) = \sigma E^2 + \epsilon[E.\frac{\partial E}{\partial t}]$$

$$H.(\nabla \times E) = H.\left(-\mu \frac{\partial H}{\partial t}\right)$$

$$H.(\nabla \times E) = -\frac{\mu}{2}\frac{\partial}{\partial t}(H.H) = -\frac{\mu}{2}\frac{\partial H^2}{\partial t}$$

$$\epsilon E.\frac{\partial E}{\partial t} = \frac{\epsilon}{2}\frac{\partial}{\partial t}(E.E) = \frac{\epsilon}{2}\frac{\partial E^2}{\partial t}$$

By substituting this in above equation,

$$-\frac{\mu}{2}\frac{\partial H^2}{\partial t} - \nabla.(\mathbf{E} \times H) = \sigma E^2 + \frac{\epsilon}{2}\frac{\partial E^2}{\partial t}$$

$$\nabla.(\mathbf{E} \times H) = -\frac{\mu}{2}\frac{\partial H^2}{\partial t} - \frac{\epsilon}{2}\frac{\partial E^2}{\partial t} - \sigma E^2$$

Taking the volume integral on both sides,

$$\int_V \nabla.(\mathbf{E} \times H)dv = -\frac{\partial}{\partial t}\int_V [\frac{1}{2}\epsilon E^2 + \frac{1}{2}\epsilon E^2]dv - \int_V \sigma E^2 dv$$

By applying the divergence theorem to the left side gives,

$$\oint_S (\mathbf{E} \times H).ds = -\frac{\partial}{\partial t}\int_V [\frac{1}{2}\epsilon E^2 + \frac{1}{2}\epsilon E^2]dv - \int_V \sigma E^2 dv$$

This equation is referred to as “Poynting Theorem”.

The cross product ***E***×***H*** is known as a pointing vector and is denoted by ‘ρ’ and units for ρ is watts/m^2

i.e $\boldsymbol{P} = (\boldsymbol{E} \times \boldsymbol{H})$

Poynting theorem states that the net power flowing out of a given volume **V** is equal to the time rate of decrease in the energy stored with in **V** minus the conduction losses.

The Pointing vectors are in the nature of power density.

The direction of flow of power is perpendicular to **E** and **H** and is in the direction of $(\boldsymbol{E} \times \boldsymbol{H})$**.**

If ***E*** and ***H*** are expressed in complex form , then we may define the complex Poyinting vector **P** as

$$\boldsymbol{P} = \frac{1}{2}(\boldsymbol{E} \times \boldsymbol{H}*)$$

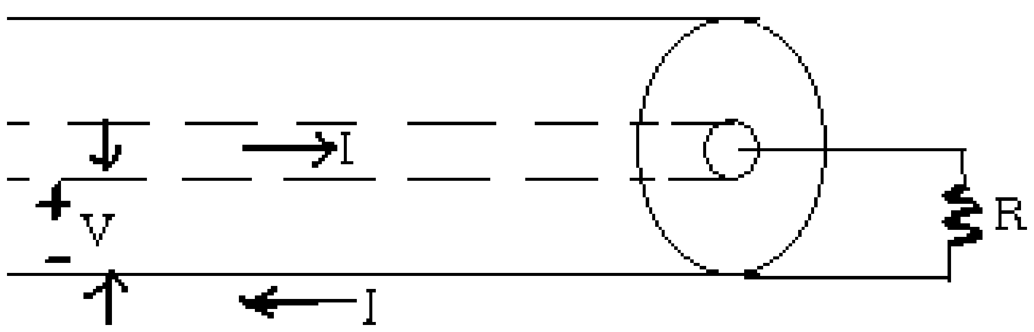
From which we may obtain the average power how per square meter as

$$Pavg = \frac{1}{2} Re(\boldsymbol{E} \times \boldsymbol{H}*)$$

Example : By interchanging the pointing vector over the cross section of co axial cable. Show that total power carried by the cable is *VI*, where *V* is the voltage and *I* is the current .

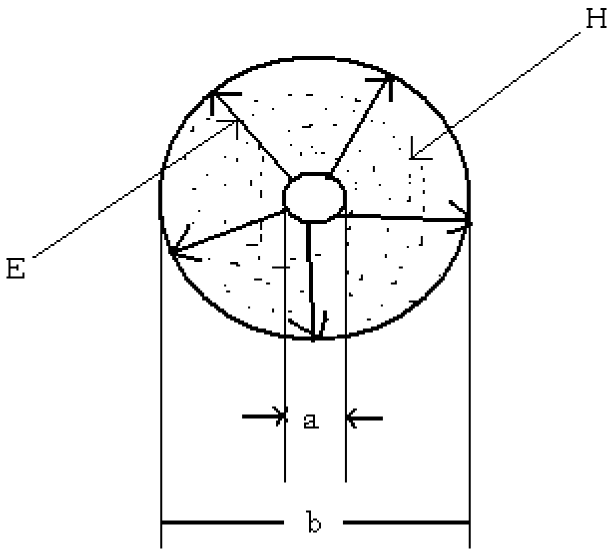
Sol : power flow in a concentric cable:

Consider the transfer of power to load resistance R along a concentric cable which has a dc voltage ***V*** between conductors and steady current ***I*** flowing in the inner and outer conductors.



The radius of inner conductors is '**a**' and the radius of outer conductor is '**b**'.

The magnetic field strength ***H*** will be directed in circles about the axis.



From Ampere's law,

$$\oint_L \mathbf{H.dl} = \ \boldsymbol{I}\mathbf{enclosed}$$

For region a$\leq \rho \leq$ b,   By applying Ampere's law,

$$\oint_L \mathbf{H.dl} = \ \boldsymbol{I}\mathbf{enclosed}$$

$$\oint_L H_\varphi i_\varphi . \rho \ d\varphi i_\varphi = I$$

$$\oint_{\varphi=0}^{2\pi} H_\varphi \ \rho \ d\varphi = I$$

$$H_\varphi . \rho \ .2\pi = I$$

$$\mathbf{H}_\varphi = \frac{\mathbf{I}}{2\pi\rho}$$

$$\mathbf{H} = \frac{\mathbf{I}}{2\pi\rho}\mathbf{i}_{\varphi} \dots\dots\dots\dots\dots\dots.(1)$$

By applying Gauss's law to an arbitrary Gaussian cylindrical surface of radius.(a< **ρ** <a).

We obtain,

$$Q = \oint_S \mathrm{D.ds}$$

$$\boldsymbol{Q = \varepsilon \oint_S E.ds}$$

$$\boldsymbol{Q = \varepsilon \oint_S E_\rho i_\rho . \rho d\varphi dz i_\rho}$$

$$\boldsymbol{Q = \varepsilon E_\rho \rho \oint_S d\varphi dz}$$

$$\boldsymbol{Q = \varepsilon E_\rho \rho . 2\pi L}$$

$$\boldsymbol{E_\rho = \frac{Q}{2\pi\varepsilon\rho L}}$$

Hence,

$$\boldsymbol{E = \frac{Q}{2\pi\varepsilon\rho L} i_\rho \dots\dots\dots\dots\dots(2)}$$

The electric field ***E*** is directed radially.

$$\boldsymbol{V = -\int_2^1 E.dl = -\int_b^a \frac{Q}{2\pi\varepsilon\rho L} i_\rho d\rho i_\rho}$$

$$V = -\frac{Q}{2\pi\varepsilon\rho L}[ln\rho]_b^a$$

$$V = \frac{Q}{2\pi\varepsilon\rho L} ln\frac{b}{a} \ldots\ldots\ldots\ldots\ldots\ldots..(3)$$

From this, $\frac{Q}{2\pi\varepsilon\rho L} = \frac{V}{ln\frac{b}{a}}$

Substituting this in equation (2) we get

$$E = \frac{V}{\rho . ln\frac{b}{a}} i_{\rho \ldots\ldots\ldots(4)}$$

The polynomial vector is,

$$P = (E \times H)$$

It is directed parallel to the axis of the cable. Since E and H are everywhere at right angles, the magnitude of **P** is simply,

$$P = (E \times H)$$

The total power flow along the cable will be given by the integration of the pointing vector over any cross-sectional surface.

$$W = \int_s (E \times H) . \mathbf{ds}$$

$$W = \int_s \frac{V}{\rho . ln\frac{b}{a}} \frac{I}{2\pi\rho} . \mathrm{i_z \rho d\rho d\varphi i_z}$$

$$W = \int_s \frac{V}{. ln\frac{b}{a}} \frac{I}{2\pi\rho} . \mathrm{d\rho d\varphi}$$

$$W = \frac{VI}{2\pi ln\frac{b}{a}} \int_a^b \frac{1}{\rho} d\rho . \int_{\varphi=0}^{2\pi} d\varphi$$

$$W = \frac{VI}{2\pi ln\frac{b}{a}} ln\frac{b}{a} * 2\pi$$

$$W = VI$$

This is the well-known result that the power flow along the cable is the product of the voltage and current.

Q) Obtain an expression for the power loss in a plane conductor in terms of the surface resistance R_s

Sol: An evaluation of the normal component of pointing vector at the surface of a conductor will give the power flow per unit area through the surface and hence the power loss in the conductor.

Let there be a tangential component of magnetic field strength at the H_{tan} surface of a metallic conductor, assumed to be infinitely large flat plate having a thickness very much greater than skin depth 'δ'.

Inside the conductor the tangential component of E is related to H_{tan} by

$$\frac{E_{tan}}{H_{tan}} = \eta$$

We know that,

$$\eta = \sqrt{\frac{jw\mu}{\sigma + jw\varepsilon}}$$

$$\eta = \sqrt{\frac{jw\mu}{\sigma}}$$

$$\eta \cong \sqrt{\frac{jw\mu}{\sigma}} L45^{\circ}$$

Where 'η'is the intrinsic impedance of the conductor.

Then the average (or real) power flow per unit area normal to the surface will be,

$$P_n(real) = \frac{1}{2} Re(E_{tan} \times H_{tan})$$

Where E_{tan} and H_{tan} are at right angles, and since for any good conductor E_{tan}leads H_{tan} by 45° in time phase then the above equation becomes,

$$P_n = \frac{1}{2} |E_{tan}||H_{tan}| cos45^{\circ}$$

$$P_n = \frac{1}{2\sqrt{2}} |E_{tan}||H_{tan}|$$

$$P_n = \frac{1}{2\sqrt{2}} \frac{|E_{tan}|^2}{\eta}$$

Where the bar'||'indicate the absolute magnitude of the complex quantity.

For a conductor which has a thickness very much greater than the skin depth' **δ** ', the surface impedance Z_s is equal to the intrinsic of the conductor.

So that,

$$P_n = \frac{1}{2\sqrt{2}} |Z_s||H_{tan}|^2 \quad watt/sq.m$$

$$P_n = \frac{1}{2\sqrt{2}} \frac{|E_{tan}|^2}{|Z_s|} \quad {watt}/{sq.m}$$

In a conductor the linear current density J_s is equal in magnitude to the tangential magnetic field strength at the surface, so

$$P_n = \frac{1}{2\sqrt{2}} |Z_s|.|J_s|^2 \quad {watt}/{sq.m}$$

Where E_{tan}, H_{tan} and J_s are peak values.

In terms of effective values,

$$P_n = \frac{1}{\sqrt{2}} \frac{|E_{t(eff)}|^2}{Z_s}$$

$$P_n = \frac{1}{\sqrt{2}} |Z_s|.|H_{t(eff)}|^2$$

$$P_n = \frac{1}{\sqrt{2}} |Z_s|.|J_{s(eff)}|^2$$

$$P_n = R_s J^2{}_{s(eff)}$$

Q) A short vertical transmitting antenna erected on the surface of a perfectly conducting earth produces an effective field strength E= 100 sinθ V/m at a distance of 1km from the antenna.Compute the Poynting vector and total power radiated.

Sol: Given,

$$E_\theta = 100 \sin\theta \ \ {v}/{m}$$

R=1km=$\mathbf{10^3}\,\boldsymbol{m}$

Assuming electro magnetic waves are propagated through free space,

$$\frac{E_\theta}{H_\emptyset} = \eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi \; or \; 377\Omega$$

$$E_\theta = 377 H_\emptyset$$

$$H_\emptyset = \frac{E_\theta}{377}$$

$$H_\emptyset = \frac{100\sin\theta}{377} \quad {}^{A}/_{m}$$

The pointing vector is given by,

$$P = E \times H = E_\theta \times H_\emptyset$$

$$P = 100\sin\theta * \frac{100\sin\theta}{377}(i_\theta * i_\emptyset)$$

$$P = \frac{10000}{377}\sin^2\theta \, i_r$$

$$P = 26.525\sin^2\theta \, i_r \; {}^{watts}/_{sq.m}$$

The total power radiated,

$$P = \int_s (E \times H).ds$$

$$P = \int_s 26.525\sin^2\theta \, i_r . . r^2\sin\theta \, d\theta d\emptyset$$

$$P = 26.525 \int_s r^2 \sin^3\theta d\theta d\emptyset$$

$$P = 26.525 * (10^3)^2 \int_{\theta=0}^{\pi} \sin^3\theta d\theta \int_{\emptyset=0}^{2\pi} d\emptyset$$

$$P = 26.525 * 10^6 * \frac{4}{3} * 2\pi$$

$$P = 222.17 \frac{Mwatts}{Sq.m}.$$

1.A perpendicularly polarized wave is incident at an angle of θ_i=15˚.It is propagating from medium 1 to medium 2 .Medium 1 is defined by ε_{r1}=8.5, μ_{r1}=1 and σ_1=0 and medium 2 is free space . IF E_i=1.0 mv/meter,determine E_r,H_i,H_r,E_t and H_t.

SOLUTION:

Given θ_i=15˚

$Medium\ 1$	$Medium\ 2$
$\varepsilon r1 = 8.5$	$free\ space$
$\mu r1 = 1$	$\varepsilon 2 = \varepsilon 0$
$\sigma 1 = 0$	$\mu 2 = \mu 0$
$\sigma 2 = 0$	

$Ei = 1.0\ mv/meter$

$\eta 1 = \sqrt{[jw\mu/\sigma + jw\varepsilon]} = \sqrt{[\mu 0/\varepsilon 0 * 8.5]} = 129\Omega$

with oblique incident on a perfect di-electric we know that

$$sin\,\theta i/sin\,\theta t\; = \sqrt{[\,\varepsilon 2\,/\,\varepsilon 1]} \qquad if\, \mu 1\; = \mu 2$$

$$sin 15^{\circ}\,/sin\,\theta t\; = \sqrt{[\,1\,/\,8.5]}$$

$$sin\,\theta t\; = \quad sin 15^{\circ} * \sqrt{8.5}$$

$$\theta t\; = 48.99^{\circ}$$

for perpendicularly polarized wave

$$Er/\,Ei\; = [[\,\eta 2 cos\,\theta i\; - \;\eta 1 cos\,\theta t]\,/[\,\eta 2 cos\,\theta i\; + \;\eta 1 cos\,\theta t]]$$

$$Er/\,Ei\;\; = 0.623$$

$$Er = 0.623 *\; Ei\; = 0.623 mV/meter$$

For perpendicularly polarized wave

$$Et/\,Ei\; = 1 + \;Er/\,Ei\;\; = 2\,\eta 2\,cos\,\theta i\,/[\,\eta 2 cos\,\theta i + \;\eta 1 cos\,\theta t]\; = 1.623$$

$$Et\quad = \;1.623 *\; Ei\;\; = \;1.623\,mv/meter$$

$$Hi\;\; = \;Ei\,/\,\eta 1 \qquad = \;7.75\,\mu\,A/m$$

$$Hr\;\; = \;Er\,/\,\eta 1 \qquad = \;4.83\,\mu\,A/m$$

$$Ht\;\; = \;Et\,/\,\eta 2 \qquad = \;4.31\,\mu\,A/m$$

2. In a perfect di-electric medium, the EM wave has maximum value of E of 10 V/m with μ_r=1 , and ε_r=4. Find the velocity of the wave , peak poynting vector, average pointing vector, impedance of the medium and peak value of the magnetic field.

SOLUTION:

Given a perfect di-electric medium

$E = 10\,V/m$

$\mu r = 1$

$\varepsilon r = 4.$

The velocity of the wave

$V = \omega/\beta$

For a perfect di- electric medium σ=0

$\beta = \omega\sqrt{\varepsilon\mu}$

$v = \omega/(\omega\sqrt{\varepsilon\mu}) = 1/\sqrt{\varepsilon\mu} = 1.5 * 10\text{^}8\, m/sec$

impedance of the medium

$\eta = \sqrt{(\mu/\varepsilon)} = 60 * \Pi$

peak value of the magnetic field

we know that E/H= η

$H = E/\eta = 10/60 * \Pi = 1/6 * \Pi\, A/m$

Instantaneous, average and complex poynting theorem:

The instantaneous power flow per square meter in terms of the electric field strength and magnetic field strength in electromagnetic field theory is given by

$P = EXH$

From the circuit theory , in similar manner in electromagnetic theory the complex pointing vector P is defined as

$$P = 1/2 * E\,XH *$$

From which we may obtain the average and reactive part of the power flow per square meter is

$$Pavg \quad = 1/2 * Re[E\,XH *]$$

$$Preactive \quad = 1/2 * Im[EXH *]$$

The product of E and H in above equation is a vector product only mutually perpendicular components of E and H contribute anything to power flow and direction of the flow is normal to the plane containing E and H

The instantaneous values of the electric and magnetic fields are

$$E = Re[E0 * e\hat{}jwt * e\hat{}j\Phi e]$$

$$E = E0 * cos(wt + \Phi e)$$

$$H = Re[H0 * e\hat{}jwt * e\hat{}j\Phi n]$$

$$H = H0\,cos(wt + \Phi n]$$

Instantaneous poynting vector is

$$P = EXH$$

$$= E0 * cos(wt + \Phi e) * H0\,cos(wt + \Phi n]$$

$$P = \frac{1}{2} E0 * H0[\cos(\Phi e - \Phi n) + \cos(2wt + \Phi n + \Phi e)] \longrightarrow \quad 1$$

Which consist of an average part and an oscillating part varies at frequency 2w, double that of E or H

The average power density

$$Pavg = 1/T \int cT \; P.dt \; \ldots\ldots\ldots\ldots\ldots.(2)$$

Where T is the time period of the time harmonic fields,

$$T = 2\Pi/\omega$$

Substituting equation (2) in (1)

$$Pavg = 1/2 \; E0 * H0 \, [\cos(\Phi e - \Phi n)] \ldots\ldots(3)$$

The average of the second term is zero

Equation (3) can also be written as

$$Pavg = 1/2 \; Re[E0 * e\wedge jwt * e\wedge j\Phi e * H0 * e\wedge - jwt * e\wedge - j\Phi n]$$

$$Pavg = 1/2 * Re[EXH *]$$

<u>Poynting theorem in complex form:</u>

Maxwell's curl equation in phasor form may be expressed as

$\nabla X H = \sigma E + jw\, \varepsilon E$

$\nabla X E = -jw\mu H$

Let us consider

$\nabla.(E\, XH *) = H *.(\nabla X E) - E.(\nabla XH *)$

$\nabla.(E\, XH *) = -jw\mu(H.H *) - \sigma(E.E *) - jw\varepsilon(E.E *)$

Taking the volume integral on both sides

$\int v\, \nabla.(E\, XH *)\, dv = -jw \int v\, (\mu\, H.H * + \varepsilon\, E.E *)\, dv - \int v\, (\sigma\, E.E *)\, dv$

By applying divergence theorem

$\int s\, (E\, XH *)\, dv = -jw \int v\, (\mu\, H.H * + \varepsilon\, E.E *)\, dv - \int v\, (\sigma\, E.E *)\, dv$

The time average densities [electric and magnetic] given by

$$Ue = 1/4 * \varepsilon E.E^*$$

$$Um = 1/4 * \mu H.H^*$$

Ex: A plane wave travelling in a medium of ε_r=1, μ_r=1 has an electric field intensity of 100√Π. Determine the energy density in the magnetic field and also the total energy density.

Sol:

Given $\varepsilon r = 1, \mu r = 1$

$E = 100\sqrt{\Pi}$

Energy density = $P = E\,XH$

$P = (E^{\wedge}2)/\eta$

$\eta = 120\,\Pi$

$P = [(100\sqrt{\Pi})^{\wedge}2\,]/(120\,\Pi) = (10^{\wedge}4)/120 = 83.3\,watt/m^{\wedge}2$

Energy stored in the magnetic field is = $1/2 * \mu H^{\wedge}2$

$\eta = E/H$

$H = E/\eta == (100\sqrt{\Pi})/(120\,\Pi)$

$WH = 1/2 * \mu(H^{\wedge}2) = (10^{\wedge} - 5)/72$

\

Reflection of uniform plane waves

Consider a uniform plane wave propagating in a certain medium at a certain velocity. As long as there is no change of medium the propagation is continuous and uninterrupted.

When an electromagnetic wave traveling in one medium impinges upon a second medium having a different dielectric constant, permeability or conductivity, the wave in general will be partially transmitted and partially reflected.

The phenomenon by which a part of the incident wave is sent back at the boundary is termed as reflection.

Reflection is effected by the characteristics of the two media.

Incident wave may be either normal or oblique.

Normal incidence

Incident wave be normal to the reflecting surface or boundary surface.

Oblique incidence

Incident wave is not normal to the boundary surface, but makes an angle such incident is termed as oblique incidence.

Reflection of uniform plane waves by a perfect dielectric – normal incidence

When a plane electromagnetic wave is incident normally on the surface of a perfect dielectric, part of energy is transmitted and part of it is reflected.

A perfect conductivity is one with zero conductivity, so that there is no loss or absorption of power in propagation through the dielectric.

Consider the plane wave traveling in the x-direction, incident on a boundary. The boundary is between two lossless dielectrics at x=0 having parameters μ_1, ε1 and μ2, ε2 respectively.

Medium 1	**medium 2**
η_1 ε_1 μ_1 σ_1	μ_2 ε_2 η_2 σ_2
Pi ----- Ei	Et
Pr ----- Er	Pt

For first medium $\eta_1 = \sqrt{(\mu 1/\varepsilon 1)}$

And for the second medium $\eta_2 = \sqrt{(\mu 2/\varepsilon 2)}$

The relationships for electric and magnetic fields are:

$$Ei = \eta 1 Hi$$

$Er = -\eta1 Hr$

$Et = \eta2 Ht$

At the dielectric, dielectric boundary the tangential components of E and H are continuous.

$$\boldsymbol{Ei + Er = Et}$$

$Hi + Hr = Ht$

$$Hi + Hr = Ht \qquad\qquad Ei + Er = Et$$

$$(Ei/\eta1) - (Er/\eta1) = Ht \qquad\qquad Ei + Er = \eta2 Ht$$

$$\frac{1}{\eta1}[Ei - Er] = Ht - - - 1 \qquad\qquad \frac{1}{\eta2}[Ei + Er] = Ht - - - 2$$

By equating equations 1 and 2 we get

$$\boldsymbol{(1/\eta1)\,[Ei - Er] = (1/\eta2)\,[Ei + Er]}$$

$Ei\,[1/\eta1 - 1/\eta2] = Er\,[1/\eta1 + 1/\eta2$

$Er/Ei = (\boldsymbol{\eta2 - \eta1})/(\boldsymbol{\eta2 + \eta1})$

The ratio $\boldsymbol{Er/Ei = (\eta2 - \eta1)/(\eta2 + \eta1)}$ is termed as reflection coefficient and it is denoted by Γ (gamma)

$$\boldsymbol{\Gamma = (\eta2 - \eta1)/(\eta2 + \eta1)}$$

The ratio Et/Ei is termed as transmission coefficient **τ**

$$\boldsymbol{\tau = Et/Ei = (Ei + Er)/Ei}$$

$$= 1 + Er/Ei$$

$$= 1 + (\eta 2 - \eta 1)/(\eta 2 + \eta 1)$$

$$= \boldsymbol{\tau} = \mathbf{2\,\eta 2/(\eta 2 + \eta 1)}$$

Similarly

$= Ei + Er = Et$ $\qquad$ $Hi + Hr = Ht$

$\eta 1 Hi + \eta 1 Hr = Et$ $\qquad$ $Hi + Hr = Et/\eta 2$

$= \eta 1\,[Hi - Hr] = Et \;-------3$ $\qquad$ $\eta 2\,[Hi + Hr]$
$= Et \;------4$

by equating 3 and 4

$$\eta 1\,[Hi - Hr] = \eta 2\,[Hi + Hr]$$

$$Hi\,[\eta 1 - \eta 2] = [\eta 1 + \eta 2]\,Hr$$

$$Hr/Hi = [\eta 1 - \eta 2]/[\eta 1 + \eta 2]$$

$$Ht/Hi = (Hi + Hr)/Hi$$

$$= 1 + Hr/Hi$$

$$= 1 + [\eta 1 - \eta 2]/[\eta 1 + \eta 2]$$

$$Ht/Hi = 2\,\eta 1/[\eta 1 + \eta 2]$$

Reflection by a perfect conductor – normal incidence

Plane wave in air incident normally upon the surface of a perfect conductor, the wave is entirely reflected.

Neither E nor H can exist within a perfect conductor so that none of the energy of the incident wave can be transmitted.

Since there can be no loss within a perfect conductor none of the energy is absorbed.

As a result the amplitudes of E and H in the reflected wave are the same as in the incident wave, and the only difference is in the direction of power flow.

Medium 1 is perfect dielectric σ1 = 0

Medium 2 is perfect conductor σ2 = infinity

$\eta 1 \ = \ \sqrt{(\, \boldsymbol{j\omega\mu 1/(\sigma 1 + j\omega\varepsilon 1)}} \ = \ \sqrt{(\boldsymbol{\mu 1/\varepsilon 1})}$

$\eta 2 \ = \ \sqrt{(\, \boldsymbol{j\omega\mu 2/(\sigma 2 + j\omega\varepsilon 2)}} \ = \ \sqrt{(\, \boldsymbol{j\omega\mu 2/\alpha})} \ = \ 0$

For electric field:

Reflection coefficient $\Gamma \ = \ Er/Er \ = \ \ [\eta 1 \ - \ \eta 2]/\,[\eta 1 \ + \ \eta 2]$

$\Gamma \ = \ 0 - \ \sqrt{(\boldsymbol{\mu 1/\varepsilon 1})}/0 \ + \ \sqrt{(\, \boldsymbol{\mu 1/\varepsilon 1})} \ = \ -1$

$\Gamma \ = \ -1$

$Er/Ei \ = \ -1$

$Er \ = \ -Ei$

Transmission coefficient τ = Et/Ei

$$Et/Ei = 2\,\eta 2/(\eta 1 + \eta 2)$$

$= 2 * 0/\sqrt{(\boldsymbol{\mu 1/\varepsilon 1})} + 0$

$= 0$

$Et = 0$

For magnetic field

Reflection coefficient

$$\Gamma = Hr/Hi = [\eta 1 - \eta 2]/[\eta 1 + \eta 2]$$

$\Gamma = \sqrt{(\boldsymbol{\mu 1/\varepsilon 1})} - 0/\sqrt{(\boldsymbol{\mu 1/\varepsilon 1})} + 0$

$= 1$

$Hr = Hi$

Transmission coefficient

$$\tau = 2\,\eta 1/(\eta 1 + \eta 2)$$

$= \tau = 2\sqrt{(\boldsymbol{\mu 1/\varepsilon 1})}/\sqrt{(\boldsymbol{\mu 1/\varepsilon 1})} + 0$

$Ht/Hi = 2$

$Ht = 2Hi$

If the expression for the electric field of the incident wave is Ei.e^-jβx

The expression for the reflected wave will be Er.e^ jβx

The tangential component of E just outside the conductor must also be zero. This requires that the sum of the electric fields strengths in the initial and reflected waves add to give zero.

Therefore Er=-Ei

The resultant electric field strength at any point a distance -x from the x=0 plane will be the sum of the fields strengths of the incident and reflected waves at that point and will be given by.

$Et(x) = Ei\, e\hat{} - j\beta x + Er\, e\hat{} j\beta x$

$= Ei\, (e\hat{} - j\beta x + e\hat{} j\beta x) \quad since\ Er = -Ei$

$= -2jEisin\beta x$

$Et(x,t) = -2jEisin\beta x.e\hat{} j\beta x$

in phase from the above equation is

$$E(x,t) = Re\,\{-2jEisin\beta x.e\hat{} j\beta x\}$$

$$E(x,t) = 2Eisin\beta x.\, sin\beta x \ ----------1$$

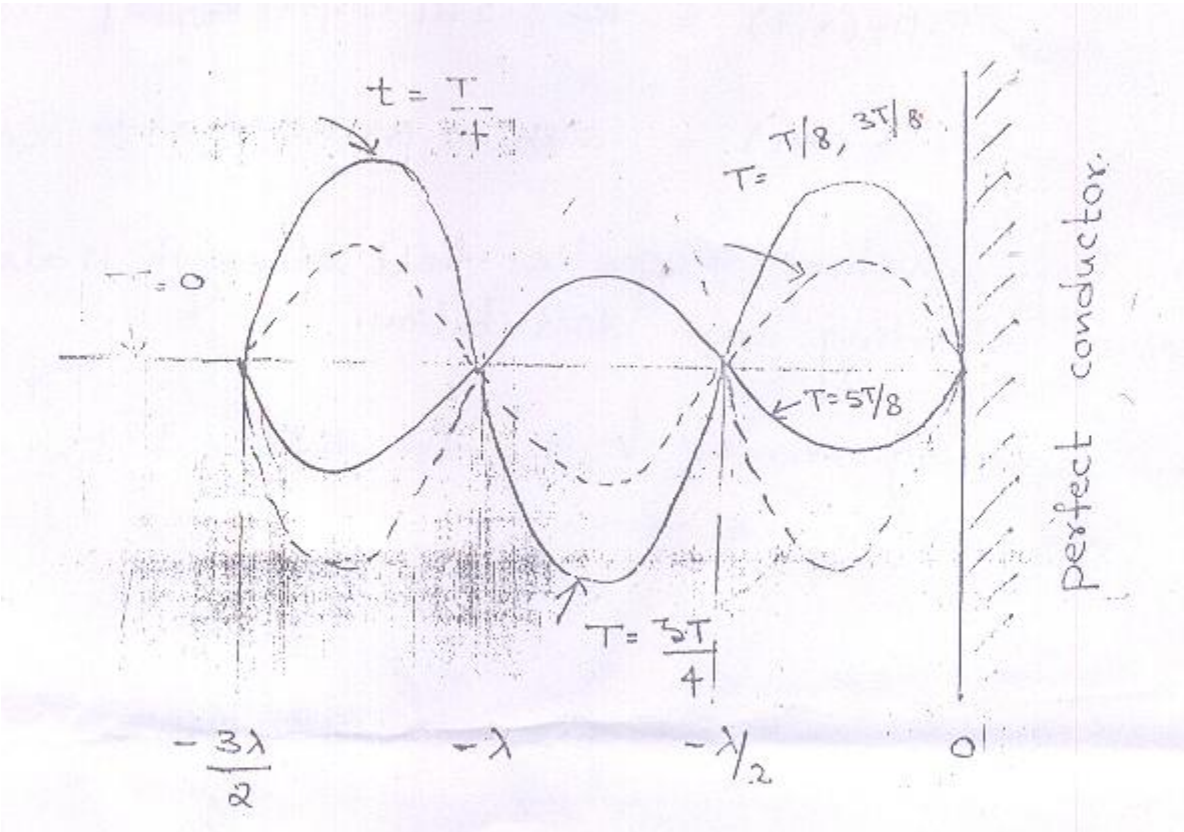
This equations show that the incident and reflected waves combine to produce a standing wave, which doesn't progress.

<u>Standing wave:</u>

When a plane wave is incident normally upon the surface of a perfect conductor, the wave gets entirely reflected. The incident and reflected waves combine together and a standing wave is formed. Since this wave does not progress, it is called standing wave.

Standing waves of E at times

$t = 0, T/8, T/4, 3T/8, T/2$ ..... are shown below



It follows that the magnetic field strength must be reflected without reversal of phase. Since if both magnetic and electric field strengths were reversed, there would be no reversal of direction of energy propagation.

The expression for the resultant magnetic field will be

$Ht(x) = Hi\, e\hat{} - j\beta x + Hre\hat{}j\beta x$

$= Hi\,[e\hat{} - j\beta x + e\hat{}j\beta x]$

$$Since\, Hr = Hi$$

$$Ht(x,t) = 2Hicos\beta x.e\hat{}j\omega t$$

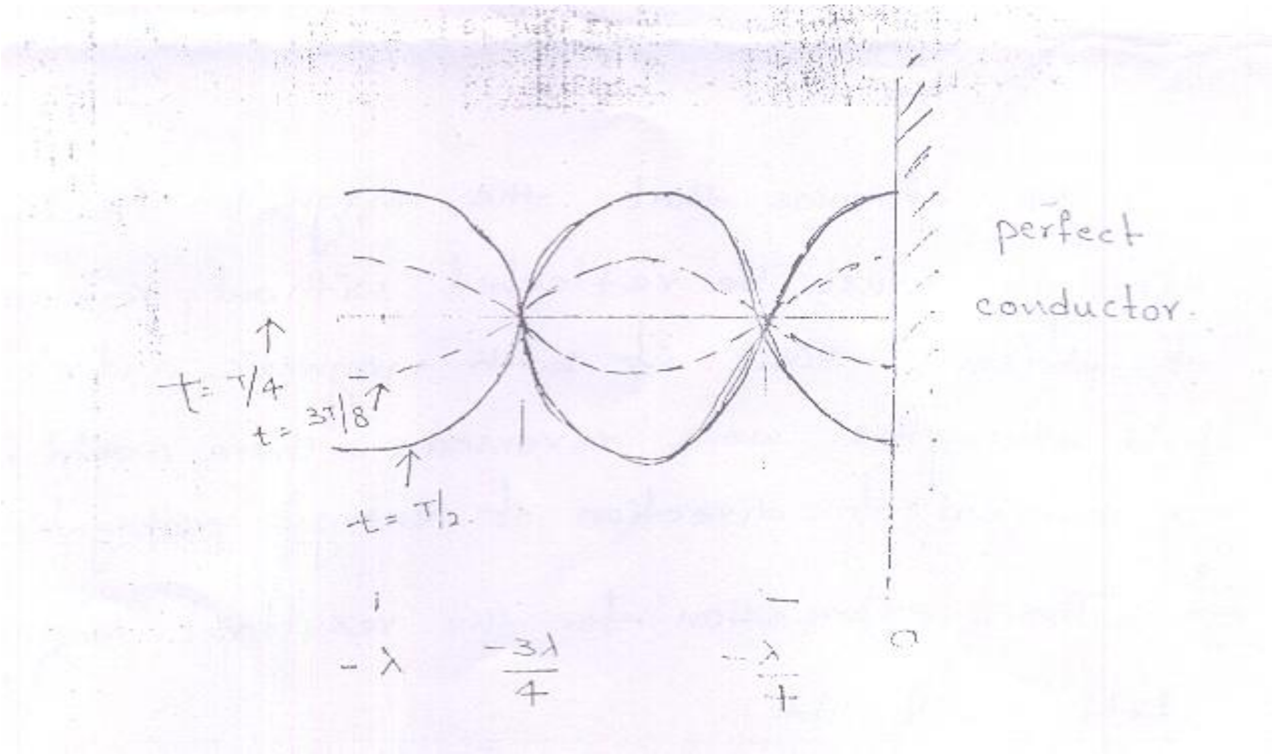
In phasor form the above equation is

$$Ht(x,t) = Re\{2Hicos\beta x.e\hat{}j\omega t\}$$

$Ht(x,t) = 2Hicos\beta x.e\hat{}j\omega t \ ------2$

The resultant magnetic field strength H also has a standing wave distribution.

Standing waves of H for different times are shown below.



An EM wave traveling in air is incident normally on a boundary between air and a dielectric having permeability same as free space and permittivity as 4. Prove that one-ninth of the incident power is reflected and eight-ninths of it is transmitted into the second medium.

Given normal incidence

Medium 1	medium 2
Air	$dielectric$
$\mu = \mu o$	$\mu = \mu o$
$\varepsilon = \varepsilon o$	$\varepsilon = \varepsilon o$

With normal incidence the reflection coefficient when a wave is incident on a dielectric is given by

$$\Gamma = Er/Ei = [\eta 1 - \eta 2]/[\eta 1 + \eta 2]$$

Transmission coefficient

$$\Gamma = Et/Ei = 2\,\eta 2/\,[\eta 1 + \eta 2]$$

$$\eta 1 = \sqrt{(\boldsymbol{\mu o}/\boldsymbol{\varepsilon o})} = 120\pi$$

$$\eta 2 = \sqrt{(\boldsymbol{\mu o}/\boldsymbol{4\varepsilon o})} = 120\pi/2$$

$$Er/Ei = (1/2 - 1)/(1/2 + 1) = -1/2/3/2 = -1/3$$

$$Er = -Ei/3$$

$$Et/Ei = (2 * 1/2)/(1 + 1/2) = 2/3$$

$$Et = 2/3\,Ei$$

$$Pi = Ei/\eta 1 = Ei/120\pi$$

$$Pr = Er/\eta 1 = \frac{\mathbf{1}}{\mathbf{9}} Ei/\eta 1$$

$$Pr = 1/9 Pi$$

$$Pt = Et/\eta 2 = \frac{\mathbf{2}}{\mathbf{3}}\,Ei/120\pi/2$$

$$Pt = \frac{\mathbf{8}}{\mathbf{9}} Ei/120\pi = 8/9 Pi$$

$$Pt = 8/9 Pi$$

Therefore one ninth of the incident power is reflected and eight-ninth of it is transformed.

Q. Find μr, εr and σ for a material in which at 100GHZ uniform plane wave α = 2 Np/m λ = 1 m and η1 = 200 ohms

SOL. In free space at 100 MHz

$$\lambda = 3\, mt$$

In this example given,

$\lambda = 1\, mt$

Hence $\sqrt{(\mu r \varepsilon r)}$ = 3 --------1

For free space $\eta = \sqrt{(\mu 0/\varepsilon 0)} = 377 ohms$

In this example given η = 200 ohms

Therefore, $\eta = \sqrt{(\mu/\varepsilon)} = 200\, ohms$

$\sqrt{(\mu o \mu r/\varepsilon o \varepsilon r)} = 200\, ohms$

$\sqrt{(\mu r/\varepsilon r)} = 200/377 = 0.5305 - - - - - - - - - - - - 2$

By multiplying equations 1 and 2 gives

$$\mu_r = 3 * 0.5305 = 1.59$$

Dividing equation 1 by 2 gives

$$\varepsilon_r = 3/0.5305 = 5.655$$

We know that $\Gamma = \sqrt{(j\omega\, \mu(\sigma + j\omega\varepsilon)}$

$\Gamma = \sqrt{(j\omega\, \mu . j\omega\varepsilon\, (1 + \sigma\, /j\omega\varepsilon)}$

$\Gamma = j\omega \sqrt{(\mu\varepsilon\, (1 + \sigma\, /j\omega\varepsilon)}$

if $\sigma/\omega\varepsilon << 1$

$$\Gamma = \sigma /2sqrt\sqrt{(\mu\varepsilon)} + j\omega\sqrt{(\mu\varepsilon)}$$

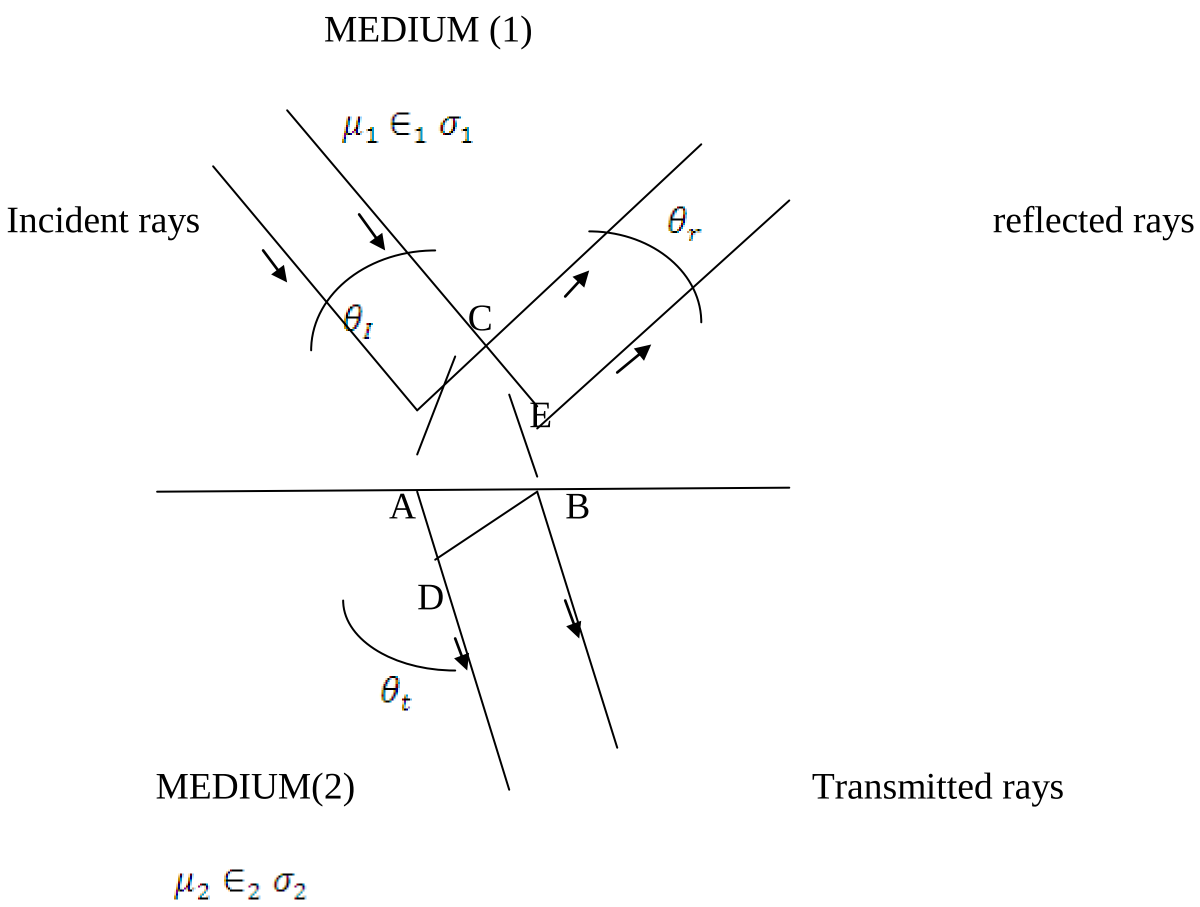
Therefore, $\alpha = \sigma /2\sqrt{(\mu\varepsilon)}$ and $\beta = j\omega\sqrt{(\mu\varepsilon)}$

$$\alpha = (\sigma /2)\,\eta$$

$\sigma = 2\alpha/ \eta = 2*2/200 = 0.02$ mhos/mt

REFLECTION BY A PERFECT DI-ELECTRIC- OBLIQUE INCIDENCE

► When ever a wave is incident obliquely on the interface between two media , part of the energy is transmitted and the part of it is reflected. But in this case the transmitted wave will be refracted .i.e the direction of propagation will be alter.



let $v_1$is the velocity of medium (1) and v_2 is the velocity of medium (2) then if the incident ray travels the distance CB where as transmitted ray travels the distance AD and reflected ray travels the distance AE

$\frac{CB}{AD} = \frac{V_1}{V_2}$……………………………(1)

CB=AE……………………………………(2)

From Δ^{le} ABC

$\sin\theta_1 = \frac{CB}{AB}$

CB=AB$\sin\theta_1$ …………………………(3)

From Δ^{le} ABD

$\sin\theta_2 = \frac{AD}{AB}$

AD=AB$\sin\theta_2$ ………………………………(4)

From Δ^{le} ABE

$\sin\theta_3 = \frac{AE}{AB}$

AE=AB$\sin\theta_3$ ………………………………(5)

By substituting equations (3) and (4) in equation (1)

We get

$$\frac{CB}{AD} = \frac{V_1}{V_2}$$

$$\frac{AB\sin\theta_1}{AB\sin\theta_2} = \frac{V_1}{V_2}$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{V_1}{V_2}$$

In terms of the constants of the medium v_1 and v_2 are given by

$$v_1 = \frac{1}{\sqrt{\mu_1 \epsilon_1}}$$

$$v_2 = \frac{1}{\sqrt{\mu_2 \epsilon_2}}$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\sqrt{\mu_2 \epsilon_2}}{\sqrt{\mu_1 \epsilon_1}}$$

If $\mu_1 = \mu_2$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\sqrt{\epsilon_2}}{\sqrt{\epsilon_1}}$$

The angle of incidence is related to the angle of reflection by the above equation which in optics is known as the laws of sine or Snell's law

By substituting (5) in (2) we get

AE=CB

$\text{AB}\sin\theta_3 = \text{AB}\sin\theta_1$

$\sin\theta_3 = \sin\theta_1$

The angle of incidence is equal to the angle of reflection

The power transmitted per square meter in a wave is the vector product of E and H

Since E and H are at right angles to each other.

The power transmitted per square meter is equal to $\frac{E^2}{\eta}$

The power in the incident wave striking AB will be proportional to $\frac{E_i^2}{\eta_1}\cos\theta_1$ and the reflected wave will be

$\frac{E_r^2}{\eta_1}\cos\theta_1$ and that transmitted through the boundary will be to $\frac{E_t^2}{\eta_2}\cos\theta_2$

By the conservation of energy

$$\frac{E_i^2}{\eta_1}\cos\theta_1 = \frac{E_r^2}{\eta_1}\cos\theta_1 + \frac{E_t^2}{\eta_2}\cos\theta_2$$

$$\frac{1}{\eta_1}\cos\theta_1 = \frac{E_r^2}{E_i^2\eta_1}\cos\theta_1 + \frac{E_t^2}{E_i^2\eta_2}\cos\theta_2$$

$$1 = \frac{E_r^2}{E_i^2} + \frac{E_t^2}{E_i^2}\frac{\cos\theta_2}{\cos\theta_1} \times \frac{\eta_1}{\eta_2}$$

$$\frac{E_r^2}{E_i^2} = 1 - \frac{E_t^2}{E_i^2}\frac{\cos\theta_2}{\cos\theta_1} \times \frac{\eta_1}{\eta_2}$$

$$\frac{E_r^2}{E_i^2} = 1 - \frac{E_t^2}{E_i^2}\frac{\cos\theta_2}{\cos\theta_1} \times \sqrt{\frac{\epsilon_2}{\epsilon_1}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\{I\}$$

Whenever a wave is incident obliquely on the interface between two mediums, it is necessary to consider separately two special cases.

CASE (I) PERPENDICULAR (HORIZONTAL) POLARISATION

In which the electric field vector is parallel to the boundary surface (or) perpendicular to the plane of incidence.

PLANE OF INCIDENCE

Plane of incidence is the plane containing the incident ray and normal to the surface.

Let the electric field strength E_i of the incident wave be in the positive ‘X’ direction and let E_r and E_t also be in the positive ‘X’ direction.

Applying the boundary conditions that the tangential component E is continuous across the boundary.

$$E_i + E_r = E_t$$

$$\frac{E_t}{E_i} = 1 + \frac{E_r}{E_i}$$

Substituting this in (I)

$$\frac{E_r^2}{E_i^2} = 1 - \sqrt{\frac{\epsilon_2}{\epsilon_1}}\{\mathbf{1} + \frac{E_r}{E_i}\}^2 \frac{\cos\theta_2}{\cos\theta_1}$$

$$\sqrt{\frac{\epsilon_2}{\epsilon_1}}\{1 + \frac{E_r}{E_i}\}^2 \frac{\cos\theta_2}{\cos\theta_1} = 1 - \frac{E_r^2}{E_i^2}$$

$$\sqrt{\frac{\epsilon_2}{\epsilon_1}}\{1 + \frac{E_r}{E_i}\} \frac{\cos\theta_2}{\cos\theta_1} = 1 - \frac{E_r}{E_i}$$

$$\frac{E_r}{E_i}\left[\sqrt{\frac{\epsilon_2}{\epsilon_1}\frac{\cos\theta_2}{\cos\theta_1}} + 1\right] = 1 - \sqrt{\frac{\epsilon_2}{\epsilon_1}\frac{\cos\theta_2}{\cos\theta_1}}$$

$$\frac{E_r}{E_i} = \frac{\cos\theta_1 - \frac{\sin\theta_1}{\sin\theta_2}\cos\theta_2}{\cos\theta_1 + \frac{\sin\theta_1}{\sin\theta_2}\cos\theta_2}$$

$$\frac{E_r}{E_i} = \frac{\cos\theta_1 \sin\theta_2 - \sin\theta_1 \cos\theta_2}{\cos\theta_1 \sin\theta_2 + \sin\theta_1 \cos\theta_2}$$

$$\gamma = \frac{E_r}{E_i} = \frac{\sin[\theta_2 - \theta_1]}{\sin[\theta_2 + \theta_1]}$$

this is the reflection co-efficient for perpendicularly polarized wave.

CASE-ii) PARALLEL (VERTICAL) POLARISATION:

In which the magnetic vector is parallel to the boundary surface or electric vector is parallel to the plane of incidence this case is often termed as vertical polarization.

Applying the boundary condition that the tangential component of E and H be continuous at the boundary.

$$E_i \cos\theta_i - E_r \cos\theta_r = E_t \cos\theta_t$$

$$(E_i - E_r)\cos\theta_i = E_t \cos\theta_t \qquad \therefore\ \theta_i = \theta_r$$

$$\theta_1 = \theta_3$$

$$\frac{E_t}{E_i} = (1 - \frac{E_r}{E_i})\frac{\cos\theta_1}{\cos\theta_2}$$

Substituting this in (I)

$$\frac{E_r^2}{E_i^2} = 1 - \left(1 - \frac{E_r}{E_i}\right)^2 \times \frac{\cos\theta_1}{\cos\theta_2} \times \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$$\left(1 - \frac{E_r}{E_i}\right)^2 \times \frac{\cos\theta_1}{\cos\theta_2} \times \sqrt{\frac{\epsilon_2}{\epsilon_1}} = (1 - \frac{E_r^2}{E_i^2})$$

$$(1 - \frac{E_r}{E_i}) \quad \frac{\cos\theta_1}{\cos\theta_2} \times \sqrt{\frac{\epsilon_2}{\epsilon_1}} \quad = 1 + \frac{E_r}{E_i}$$

$$\frac{E_r}{E_i} = \frac{\sqrt{\epsilon_2}\cos\theta_1 - \sqrt{\epsilon_1}\cos\theta_2}{\sqrt{\epsilon_2}\cos\theta_1 + \sqrt{\epsilon_1}\cos\theta_2}$$

$$\frac{E_r}{E_i} = \frac{(\sqrt{\epsilon_2} \div \sqrt{\epsilon_1}) \times \cos\theta_1 - \cos\theta_2}{(\sqrt{\epsilon_2} \div \sqrt{\epsilon_1}) \times \cos\theta_1 + \cos\theta_2}$$

$$\frac{E_r}{E_i} = \frac{(\sin\theta_1\cos\theta_1 - \sin\theta_2\cos\theta_2)}{(\sin\theta_1\cos\theta_1 - \sin\theta_2\cos\theta_2)}$$

$$\frac{E_r}{E_i} = \frac{(\sin 2\theta_1 - \sin 2\theta_2)}{(\sin 2\theta_1 - \sin 2\theta_2)}$$

This equation is the reflection co-efficient for parallel polarization

<u>BREWSTER ANGLE:</u>

Brewster angle is defined as the angle at which there is no reflected wave when the parallel polarized wave is incident on a perfect dielectric oblique

$$\frac{E_R}{E_I} = (\sqrt{\epsilon_2}\, cos\theta_1 \sqrt{}\epsilon_1 \cos\theta_2)/\sqrt{\epsilon_2}\, cos\theta_1 + \sqrt{}\epsilon_1 \cos\theta_2 = \gamma$$

$E_R = 0$

$$\sqrt{\epsilon_2}\, cos\theta_1 - \sqrt{\epsilon_1} \cos\theta_2 = 0$$

$$\sqrt{\epsilon_2}\, cos\theta_1 = \sqrt{\epsilon_1} \cos\theta_2$$

$$\sqrt{1 - sin^2\theta_1} = \sqrt{\epsilon_1}/\sqrt{}\epsilon_2\, (\sqrt{1 - sin^2\theta_2})$$

$$1 - sin^2\theta_1 = \epsilon_1/\epsilon_2\, (1 - \epsilon_1/\epsilon_2\, sin^2\theta_1)$$

$$\{\left(\frac{\epsilon_1}{\epsilon_2}\right)^2 - 1\}\, sin^2\theta_1 = \frac{\epsilon_1}{\epsilon_2} - 1$$

$\{(\epsilon_1^2 - \epsilon_2^2)/\epsilon_2^2\} sin^2\theta_1 = (\epsilon_1 - \epsilon_2)/\epsilon_2$

$$\{(\epsilon_1 + \epsilon_2)/\epsilon_2\} sin^2\theta_1 = 1$$

$sin^2\theta_1 = \epsilon_2/\epsilon_1 + \epsilon_2$

$$\sin\theta_1 = \sqrt{\epsilon_2/(\epsilon_1+\epsilon_2)}$$

$$cos^2\theta_1 = 1 - sin^2\theta_1$$

$$= 1 - (\epsilon_2/(\epsilon_1+\epsilon_2))$$

$$cos^2\theta_1 = \epsilon_1/\epsilon_1+\epsilon_2$$

$$tan^2\theta_1 = (\epsilon_2/\epsilon_1)$$

$$\tan\theta_1 = \sqrt{\epsilon_2/\epsilon_1}$$

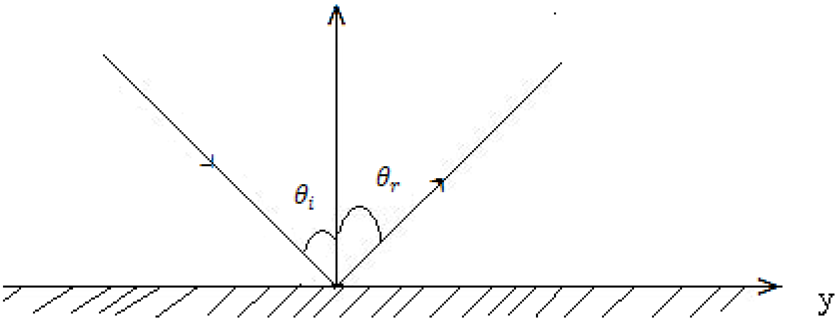
$$\tan\theta_B = \sqrt{\epsilon_2/\epsilon_1}$$

$$\theta_B = tan^{-1}\sqrt{\epsilon_2/\epsilon_1}$$

This is Brewster angle with parallel polarization with oblique incidence on a perfect dielectric medium

Reflection of uniform plane waves with oblique incidence by a perfect conductor

If a wave is incident in a perfect conductor, the wave is totally reflected with the angle of incidence equal to the angle of reflection.



Case I: perpendicular polarization:

$$E_{incident} = E_i e^{-j\beta[y\sin\theta - z\cos\theta]}$$

$$E_{reflected} = E_r e^{-j\beta[y\sin\theta + z\cos\theta]}$$

$\therefore$ The total electric field strength will be

$$E = E_{incident} + E_{reflected}$$

$$E = E_i e^{-j\beta[y\sin\theta - z\cos\theta]} + E_r e^{-j\beta[y\sin\theta + z\cos\theta]}$$

$$E = E_i [e^{-j\beta[y\sin\theta - z\cos\theta]} - e^{-j\beta[y\sin\theta + z\cos\theta]}]$$

$$\because \quad E_i = -E_r$$

$$E = 2j.E_i \sin(\beta z\cos\theta) e^{-j\beta y\sin\theta}$$

Parallel polarization:

$$H_{incident} = H_i e^{-j\beta[y\sin\theta - z\cos\theta]}$$

$$H_{reflected} = H_r e^{-j\beta[y\sin\theta + z\cos\theta]}$$

$$H_i = H_r$$

The magnitude field strength vector H will be reflected without phase reversal

$$E = 2jE_i.\cos(\beta z\cos\theta)e^{-j\beta y\sin\theta}$$

Total Internal Reflection:

If 'ἐ1' is greater than 'ἐ2' both the reflection coefficients become complex numbers when

$$\sin\theta_1 > \sqrt{\frac{\varepsilon 2}{\varepsilon 1}}$$

Both the reflection co-efficient take on the form ($\frac{a+jb}{a-jb}$) and thus have the unit magnitude.

In other words, the reflection is total provided that θ_1 is great enough provided that medium 1 is denser than medium 2

The reflection coefficients are

$$\frac{E_r}{E_i} = \frac{\sqrt{\varepsilon 1}\cos\theta_1 - \sqrt{\varepsilon 2}\cos\theta_2}{\sqrt{\varepsilon 1}\cos\theta_1 + \sqrt{\varepsilon 2}\cos\theta_2}$$ for perpendicular polarization

$$\frac{E_r}{E_i} = \frac{\sqrt{\varepsilon 2}\cos\theta_1 - \sqrt{\varepsilon 1}\cos\theta_2}{\sqrt{\varepsilon 2}\cos\theta_1 + \sqrt{\varepsilon 1}\cos\theta_2}$$ for parallel polarization

For total internal reflections the value of reflection coefficients becomes as

$$\frac{E_r}{E_i} = \frac{\cos\theta_1 + j\sqrt{\sin^2\theta_1 - (\frac{\varepsilon 2}{\varepsilon 1})}}{\cos\theta_1 - j\sqrt{\sin^2\theta_1 - (\frac{\varepsilon 2}{\varepsilon 1})}}$$ for perpendicular polarization

$$\frac{E_r}{E_i} = \frac{\frac{\varepsilon 2}{\varepsilon 1}\cos\theta_1 + j\sqrt{\sin^2\theta_1 - (\frac{\varepsilon 2}{\varepsilon 1})}}{\frac{\varepsilon 2}{\varepsilon 1}\cos\theta_1 - j\sqrt{\sin^2\theta_1 - (\frac{\varepsilon 2}{\varepsilon 1})}}$$ for parallel polarization

Critical angle:

In the condition of total internal reflection, if $\sin\theta_1 = \sqrt{\frac{\varepsilon 2}{\varepsilon 1}}$ then, that particular incident angle is known as "critical angle"

$$\therefore \sin\theta_c = \sqrt{\frac{\varepsilon 2}{\varepsilon 1}}$$

$$\theta_c = \sin^{-1}\left(\sqrt{\frac{\varepsilon 2}{\varepsilon 1}}\right)$$